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The Art of Teaching Arithmetic

Rev. John Hunter

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A Manual for Schoolmasters

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THE ART OF TEACHING ARITHMETIC

A Manual of Method for Schoolmasters,

CONTAINING

QUESTIONS ON METHOD, WITH ANSWERS; AND NUMEROUS PROBLEMS, WITH SOLUTIONS.

DESIGNED TO FACILITATE AND IMPROVE
THE TEACHING OF ARITHMETIC, AND TO PREPARE NORMAL STUDENTS, ETC.,
FOR THE GOVERNMENT EXAMINATIONS IN THAT DEPARTMENT
OF INSTRUCTION.

BY THE REV. J. HUNTER, A.M.,

FORMERLY VICE-PRINCIPAL OF THE TRAINING COLLEGE, BATTERSEA.

WITH AN APPENDIX OF WORKED SPECIMENS OF
EXAMINATION PAPERS SET BY THE SOCIETY OF ARTS, AND THE
GOVERNMENT EDUCATION DEPARTMENT.

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PREFACE.

THE object of this little work is sufficiently described on the title-page, and the expediency of its publication will, it is believed, be very readily admitted by those who are professionally interested in the requirements by which Government seeks to ascertain, and is enabled to certify, scholastic ability.

The Questions on Method which are here proposed, include the principal varieties proposed from time to time in the Arithmetical Department of the Government Examination Papers; and the answers given are, for the most part, intended as specimens of what should be considered appropriate and sufficient on the part of those examined. In some instances, however, as in that of Equation of Payments, the answer is more minute and extensive than the time devoted to examination would allow to be imitated,—opportunity having been taken, in such cases, to communicate useful information, in addition to the sufficiency exacted by the occasion.

On the whole, it is hoped that the specimens of Arithmetical Explanation and Solution presented in the following pages will be found to succeed, in good measure, in promoting the useful purposes for which the work is designed.

THE

ART OF TEACHING ARITHMETIC.

PART FIRST.

QUESTIONS ON METHOD.

1. What utility should be aimed at in teaching Arithmetic to children?
2. State some general principles on which the successful teaching of Arithmetic depends.
3. At what stage, and in what manner, would you commence the teaching of Arithmetic? What kind of lessons are suitable for a class of children learning the first principles of Notation?
4. How might a printed treatise on Arithmetic be occasionally employed with advantage as a class reading-book?
5. Explain the practicability and the advantage of combining simultaneous and individual instruction in Arithmetic.
6. Having prescribed a testing exercise in Arithmetic to be worked by a class on their slates, what use can you make of the black board in turning such exercise to good account?

7. What expediency should be kept in view, and what inexpediency avoided, in using the black board for arithmetical instruction?
8. Why does Arithmetic generally require a classification of children different from that which is suitable for other departments of class instruction? How would you meet this requirement?
9. What method might be employed to prevent copying in Arithmetic, when the children cannot be widely separated?
10. What caution is necessary in communicating what are called "short methods" of calculation?
11. When numbers become higher in value than *nine*, we express them with tens and units;—Apply this principle of notation to explain the addition of 369, 5073, 7409, and 874. What methods of verification may be employed?
12. Explain the natural method, and the usual artificial method, of subtracting 749 from 927. Illustrate the artifice of borrowing, by adding 62 to each number before subtracting. Why is the uniform addition of *ten* preferable?
13. Subtract 509962 pence from 809035 pence, and apply some method of verification. Shew that both in Addition and Subtraction the given quantities must be used in one denomination.
14. Make it appear that multiplication shortens a process of addition.
15. Shew why multiplying by 72 gives the same result as multiplying successively by 8 and 9.

16. Explain how 5 shillings multiplied by 3 gives the same result as 3 shillings multiplied by 5.
17. Explain simply why multiplying by 10, 100, &c., requires merely to annex one, two, &c., ciphers to the multiplicand; and thence derive the mode of multiplying by 30, 400, &c.
18. Explain each step in the process of multiplying 47302 by 7052.
19. Apply to the preceding example the method of verification called "casting out the nines." How would you demonstrate that method? Shew that it is only a presumptive test; and specify any other method of verifying the process of multiplication.
20. Shew that a multiplier is always an abstract number;—Illustrate by the calculation of the wages of 6 men for 4 days at 3 shillings each man per day.
21. What means would you recommend for facilitating the recollection of the Multiplication Table? Which of its products do children generally find greatest difficulty of recollecting?
22. Explain the process of dividing 30827 days by 9 days; ditto by 59 days. How would you explain to an elementary class the nature of the remainder?
23. Give the two-fold definition of Division, and exemplify by explaining, first, the division of 36 ounces by 4 ounces; secondly, that of 36 ounces by 4. What quantity, strictly considered, is the quotient in the latter example?

24. Which of the quantities in Division is always properly to be regarded as abstract? How would you verify the process of division?

25. Explain what is called the Italian method of Long Division.

26. Divide 3760 successively by 6 and 9, and demonstrate the usual method of determining the entire remainder from the two partial remainders.

27. Explain why 48 multiplied by 6 gives the same result as the *third part* of 48 multiplied by *three times* 6.

28. Shew why 48 divided by 6 gives the same result as the *third part* of 48 divided by the *third part* of 6; and prove that, universally, a quotient is not altered by previously multiplying or dividing both dividend and divisor by any one number.

29. A quantity is to be multiplied by 12, and the product to be divided by 18;—Why is the result the same if we divide the given quantity by 18, and multiply the quotient by 12? Which order of operation is generally preferable, and why? Shew why multiplying by 2 and then dividing by 3 gives the same result.

30. Account for the abbreviations £ *s.* *d.*, *Lb.*, *Cwt.*, *Dwt.*

31. Explain the reduction of £53 to shillings; distinguish the value of the result from that of £53 × 20.

32. Reduce by an explanatory process, 5 cwt. 3 qrs. 9 lbs. to lbs.; also 4694 farthings to £ *s.* *d.* Verify the work.

33. By what steps would you assist a child to answer correctly such questions as the following;—(*a*) How many pence are in 39 half-crowns? (*b*) How many feet in $47\frac{1}{2}$ yards?

34. Explain the conversion of 136 yards to English ells; and of 117524 square inches to the higher denominations.

35. State the extent of resemblance between Compound and Simple Addition.

36. Explain each step in the process of subtracting £12, 13s. $7\frac{1}{2}$d. from £58, 11s. $7\frac{1}{4}$d.; also, in calculating the 78th part of £239, 17s. 5d.

37. Give an explanatory solution of the question,—How often does a coach-wheel, 11 feet in circumference, turn round in passing over a mile?

38. If 1 yard cost 3s. 6d. what cost 7 yards? If 9 yards cost 31s. 6d., what cost 1 yard? Derive from these respective examples of multiplication and division a question of the form usual in Simple Proportion, and work the question by first principles. What is here meant by first principles?

39. Proportion is either direct or inverse: Explain; Construct an example of each kind.

40. Give general directions for solving by first principles a question in Simple Proportion.

41. If 17 yards cost 43s., what do 13 yards cost? In working this by the method of "statement," you would multiply the 3rd term by the 2nd; what is the *commercial* meaning of that process? Give a reason for the subsequent division by the 1st term.

42. How many acres can 16 men reap in the same time that 23 men take to reap 15 acres? Explain the working of this sum, having recourse to no other than the first four rules of Arithmetic.

43. Define a fraction. Distinguish between a proper and an improper fraction.

44. Employ the expression $\frac{3}{4}$ to illustrate the meaning of a fraction. Shew by a diagram, or otherwise, that it may mean either 3 fourths of 1 whole, or 1 fourth of 3 wholes. To which of these interpretations of a fraction are the names Numerator and Denominator properly applicable?

45. Shew that $\frac{5}{7}$ is equal to $\frac{30}{42}$.

46. Why must fractions have a common denominator before they can be added or subtracted?

47. Prove the rule for the multiplication of Vulgar Fractions.

48. Prove the rule for the division of Vulgar Fractions.

49. Express $\frac{2}{3}$ of $\frac{5}{8}$ by using the common sign of multiplication. By what fraction of $\frac{2}{3}$ must the product be increased to make unity?

50. Shew that multiplying a number by 7 has the same effect as dividing the number by $\frac{1}{7}$

51. Shew that dividing the numerator of a fraction by any number gives the same result as multiplying the denominator by that number; and that multiplying the numerator is equivalent to dividing the denominator.

52. One quantity is to another as 8 to 3; what name is

given to this kind of relation? What does it mean?

53. Proportion is sometimes defined to be an equality of ratios; is this strictly accurate? Give an explanatory answer.

54. The product of the extremes of a proportion is also that of the means. Explain and account for this by the example—How many men require 8 days, when 12 men take 6 days?

55. Work by first principles, and also by proportion statement, with explanation, the following:—How many yards of cloth, at 5s. 6d. a yard, should be given for 19 lbs. of tea at 3s. 8d. a lb.?

56. If 21 articles cost £35, how many of them cost £20? Imitate the simplicity of form which this question presents, by changing the language of the following:—

(*a*) By a payment of £74, 11s. I become entitled to £5, 10s. per annum; what income should I derive from disposing of £546, 14s. in the same way?

(*b*) What should the 7d. loaf weigh when wheat costs 52s. a quarter, supposing it ought to weigh 39 ounces when the price of a quarter of wheat is 48s.?

(*c*) A cistern, whose capacity is 1000 gallons, is supplied with water by a pipe which pours into it 8 gall. 2 qts. per minute. By a leakage, 1 gall. 2 qts. 1 pt. are lost every minute during the time of filling. In what time will the cistern be filled?

57. If 6 men do a work in 10 days, how many men would do the same in 15 days? Change this ques-

tion to one in Compound Proportion, by the introduction of other ratios affecting the times and the agents. What do you understand by a compound ratio?

58. Give general directions for working, by first principles, a question in Compound Proportion. Work by first principles, and also by statement, the following:—

(*a*) If I get 7 dozen apples for 10s. 5d., when 16 apples weigh 5 lbs.; how many, at the same price per lb., but of which 10 apples weigh 3 lbs., should I get for 20d.?

(*b*) If 20 oxen, in 7 weeks, eat $17\frac{1}{2}$ weeks' growth of 6 acres of grass; of how many acres will 35 oxen eat $22\frac{1}{2}$ weeks' growth in 12 weeks?

59. What is meant by saying that one quantity is $\frac{3}{8}$ of another? A.'s share of a sum of money is $\frac{3}{8}$ of B.'s; what fraction of their joint amount has each, and what fraction of B.'s share equals the excess of B.'s above A.'s?

60. Exemplify the steps by which you would demonstrate the process of finding the greatest common measure of 27 and 72.

61. Establish the rule that the product of two numbers divided by their greatest common measure gives their least common multiple.

62. Demonstrate the simplest mode of subtracting $25\frac{4}{9}$ from $32\frac{1}{3}$.

63. How would you explain to children the addition of $\frac{3}{4}$ of a shilling and $\frac{1}{6}$ of half-a-crown?

64. Demonstrate the process of converting $\frac{5}{7}$ of an English ell into the fraction of a yard.

65. Two numbers are in the ratio of 7 to 9; what fraction of their sum is their difference?

66. Distinguish Vulgar and Decimal Fractions, and shew why, in converting $\frac{5}{8}$ to a decimal fraction, you annex ciphers to the numerator and divide by 8.

67. How would you teach the notation of decimal fractions? Apply your method to explain the decimal coinage of sovereigns, florins, cents, and mils.

68. Shew that £·475 equals 47 cents 5 mils, and also equals 475 mils. Explain the notation of the expression £·57$\frac{1}{4}$, and alter it to the purely decimal form.

69. Shew, by the example 51·2407—8·93146, how a sum in subtraction of decimals should be written out so as to exhibit the reason of every step in the working of it.

70. Whether is ·067 or ·068 more nearly equal to ·06748? What is the defect in one case, and what the excess in the other?

71. Prove the rule for pointing (*a*) in multiplication of decimals, (*b*) in division of decimals.

72. If 54$\frac{5}{8}$ yards cost £8, 12s. 6d., what cost 14$\frac{1}{4}$ yards? Change the subdivisions into decimal fractions of a yard and a £, and then work the question by first principles.

73. What kind of vulgar fractions can be converted into terminate decimals? Prove ·262626, &c., equal to $\frac{26}{99}$.

74. Shew from first principles how to calculate values by Practice.

75. What is meant by a per centage? Explain the calculation of 4 per cent. on £625, and find what sum calculated at 5 per cent. produces the same result.

76. Express in its simplest form the fraction indicated by $3\frac{1}{3}$ per cent. Having bought a quantity of goods, I sold them forthwith at a profit of $3\frac{1}{3}$ per cent.; what portion of the selling price was gain? and what portion of the selling price was the buying price?

77. Express in simplest terms the ratio of the population of a parish, ten years ago, to its present population, the increase during the interval having been $2\frac{1}{2}$ per cent.

78. Express in the usual elementary form of questions in Simple Proportion the following:—

(*a*) Find the profit on £280 at $5\frac{1}{2}$ per cent.

(*b*) Find the amount of £56 when increased by a gain of 3 per cent.

(*c*) At what rate per cent. loss is £60 reduced to £52?

(*d*) What sum should yield 5s. 6d. at $\frac{3}{8}$ per cent?

(*e*) By selling goods at 12s. 10d. I gained $2\frac{2}{3}$ per cent.; what did they cost me?

(*f*) I cleared $3\frac{1}{2}$ per cent. by selling cloth at 5s. 9d. a yard; what should I have cleared per cent. by selling at 5s. 10d.?

(*g*) If by selling goods at 7s. 4d. I gained $4\frac{1}{2}$ per cent., by what selling price should I have gained £6, 17s. 6d. per cent.?

79. Shew that 5 per cent. per annum for 4 years is equivalent to 4 per cent. per annum for 5 years. What is the equivalent rate per cent. per annum for 1 year, and what the equivalent time for 1 per cent. per annum?

80. If an investment of 8d. become $12\frac{1}{2}$d. in 9 months, what investment will become $13\frac{3}{4}$d. in 6 months? Work this by first principles. What mistake are many pupils likely to make in attempting to solve the problem?

81. Calculate, by the method of Practice, $4\frac{1}{2}$ per cent. on £1350. Enunciate in precisely the same form, and solve in the same way, the following question: —Find the simple interest on £156 for $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent. per annum.

82. Find, by first principles, (*a*) the rate per cent. per annum, simple interest, to make £7 amount to £7, 17s. 6d. in 4 years; (*b*) the time required to make £15 yield £3, 7s. 6d., simple interest, at 5 per cent. per annum; (*c*) the principal that will amount to £513 in $3\frac{1}{2}$ years at 4 per cent. per annum, simple interest.

83. By the ordinary method of computing the simple interest on £230 for 7 years at $3\frac{1}{2}$ per cent. per annum, you would first multiply by $3\frac{1}{2}$, then by 7, or first by 7, then by $3\frac{1}{2}$; what *commercial* meaning can you assign to each of these processes?

84. Give notes of a lesson on the common process of calculating simple interest for days.

85. Explain the amount of £75 for 4 years at 5 per cent. per annum, compound interest, as equal to $1{\cdot}05^4 \times 75$ pounds.

86. What is the true present worth of a sum of money due at a future time? Distinguish between true discount and bankers' discount.

87. What fraction, of any sum of money, is the true discount on that sum for 9 months at 5 per cent. per annum, simple interest?

88. A sum of money is payable 3 years hence. What fraction of that sum is its true present value, reckoning money worth $6\frac{1}{4}$ per cent. per annum, simple interest?

89. Give the reason for each step in the process of calculating the true discount on £88, 3 florins, 5 cents, 6 mils, due 63 days hence, at 4 per cent. per annum, simple interest.

90. Explain the process of computing the true present value of £207, 6s. $1\frac{3}{5}$d. due 2 years hence, at 4 per cent. per annum, compound interest.

91. There are three numbers in the proportion of 5, 9, and 6; explain the relation here stated, and demonstrate that the first number must be a third of the sum of the other two.

92. If A : B :: 5 : 6, and B : C :: 9 : 10, what three numbers will most simply express the continuous proportional relation of A, B, and C? And, if the sum of A and C is 140, what is the difference of A and B?

93. Compare the different methods usually given for cal-

culating the equation of payments, and point out the true source of the apparent anomaly.

94. Exhibit the involution of 57 to the second power, in an arithmetical form suitable for a deduction of the method of extracting the square root of 3249, and perform the extraction accordingly. Explain the method of pointing in extraction of the square root.

95. What do you understand by the decimal or denary scale of notation? Illustrate by means of the examples 8325 and 763·5248.

96. What is the septenary scale? Write out the first twelve consecutive numbers in that scale.

97. Exhibit the steps of the process by which you would transform the expression 2244 in the quinary scale to an equivalent expression in the undenary scale.

98. Explain the steps of the process of extracting the square root of the quinary number 2244, conforming to the quinary scale throughout the operation.

PART SECOND.

PROBLEMS.

1. A mason reduced the size of a stone, so as to leave five times as much as he cut from it; the stone originally measured $7\frac{1}{2}$ cubic feet. Required its reduced bulk.

   Ans. $6\frac{1}{4}$ cubic feet.

2. The net weight of a quantity of goods was 2 cwt. 46 lbs., an allowance of 16 lbs. per cwt., tare, having been deducted; what was the gross weight?

   Ans. 2 cwt. 91 lbs.

3. Bought a cask of oil for £26, 19s.; for what must I sell it, that $\frac{2}{9}$ of the selling price may be profit?

   Ans. £34, 13s.

4. A sum of money is divided between M and N, and N's share is equal to $\frac{5}{11}$ of the difference between his own share and that of M. What portion of the whole has each?

   Ans. M $\frac{16}{21}$, N $\frac{5}{21}$.

5. Two men shoot by turns at pigeons let out of a trap; the one hits 3 birds out of 5, while the other hits 5 out of 8; how many shots must each have, to kill 98 between them?

   Ans. 80.

6. The sum of £1647, 9 florins, 4 cents, is due from

London to Paris, when £1 is worth 25 francs; what sum should be remitted if 11 florins are worth 27 francs?

Ans. £1678, 4 fl., 5 cts., 7 mls.

7. A railway train, having performed a journey at $\frac{3}{5}$ of its proper speed, arrived at the termination $2\frac{1}{2}$ hours behind time; in what time should the journey have been performed?

Ans. $3\frac{3}{4}$ hours.

8. If $1\frac{1}{2}$ lb. of tea be worth 5 lbs. of coffee, and $5\frac{1}{2}$ lbs. of coffee worth 13 lbs. of sugar, and $7\frac{1}{2}$ lbs. of sugar worth $8\frac{1}{4}$ lbs. of soap, how many lbs. of soap are 4 lbs. of tea worth?

Ans. $34\frac{2}{3}$ lbs.

9. Divide £8, 3 florins, 7 cents, among four men, giving the first $1\frac{1}{2}$ as much as the second, the second $1\frac{1}{4}$ as much as the third, and the third $1\frac{1}{6}$ as much as the fourth.

Ans. 1st, £3, 1 fl., 5 cts., &c.

10. Of the adult male population of a village, the proportion of those who are labourers is 11 in 20, tradesmen 7 in 24, professional men 9 in 80, and there are, besides, 33 gentlemen. How many male adults in all?

Ans. 720.

11. One vessel contains 11 pints of rum mixed with 13 pints of water; another vessel contains 8 pints of rum mixed with 10 pints of water; what are the comparative strengths of the mixtures?

Ans. 33 and 32.

12. Two men or five boys can perform a work in 7 days; how many men, along with three boys, could perform it in $3\frac{1}{3}$ days?

Ans. 3.

13. Two third-class passengers, for the same journey, are charged 3s. 4d. and 8s. 4d., respectively, for excess of luggage; had the luggage all belonged to one of them, he would have been charged for excess 35s. How much would they have been charged if no luggage had been allowed free?

Ans. 26s. 8d. and 31s. 8d.

14. A mercer sold 54 yards of cloth for £6, 10s., his profit being the cost of $7\frac{1}{2}$ yards; what was his gain per cent.?

Ans. $13\frac{8}{9}$ per cent.

15. If 4 per cent. be gained on the prime cost of an article, how much per cent. of the selling price is profit?

Ans. $3\frac{11}{13}$ per cent.

16. What would be gained or lost per cent. by paying $10\frac{1}{2}$d. for that whole number of mils which is nearest it in value?

Ans. $\frac{4}{7}$ per cent. gain.

17. If a wholesale dealer sell to a retail dealer at 8 per cent. profit, and the retailer sell to the consumer at 20 per cent. profit, what proportion of the price paid by the consumer is profit?

Ans. $\frac{37}{162}$.

18. The value of 20 lbs. of sugar increased by 4 per cent., is equal to the value of 3 lbs. of tea dimin-

ished by 1 per cent.; how many lbs. of tea are equivalent to 26 lbs. of sugar?

Ans. 3 lbs. $11\frac{2}{3}$ oz.

19. A tradesman raises the price of an article from 13d. to 16d., and finds that his total receipts thereby are only 10 per cent. greater than before. Find the decrease per cent. in the quantity he sells.

Ans. $10\frac{5}{8}$ per cent.

20. If brandy is bought at 25s. a gallon, at what price must it be sold to clear as much on £100 as 5 gallons are sold for?

Ans. 26s. 8d.

21. A hat which cost 10s. will last 7 months, and one which cost 18s. will last 13 months; which is the cheaper? What would be saved at the end of 3 years, by wearing the cheaper kind of hat rather than the other, allowing simple interest at 5 per cent. per annum?

Ans. 1s. $0\frac{1}{2}$d. nearly.

22. What principal improved for 3 years, at 4 per cent. per annum, would amount to £1, 18s. more by compound than by simple interest?

Ans. £390, 12s. 6d.

23. Suppose a railway train, proceeding at the rate of $\frac{2}{3}$ of a mile in a minute, to be audible at the distance of $2\frac{1}{4}$ miles; how long exactly will its noise precede it—sound travelling at the rate of 1130 feet per second?

Ans. $3\frac{903}{4520}$ minutes.

24. In every 24 ounces of what is called gold, only 18

ounces in jewellery, and 22 ounces in sovereigns, are really gold. The weight of a sovereign is $123\frac{1}{6}\frac{7}{2}\frac{1}{3}$ grains. If a goldsmith sells chains for their weight in sovereigns, what does he charge per ounce for the workmanship?

Ans. 14s. $1\frac{1}{1}\frac{0}{1}$d.

25. At a certain election, 350 persons voted, and the successful candidate had a majority of 26 votes. How many voted for him?

Ans. 188.

26. Divide 12 florins 3 cents between a brother and a sister, giving the latter $5\frac{1}{2}$ cents less than the former.

Ans. Brother, 6 fl. 4 cts. $2\frac{1}{2}$ mls.

27. A rectangular field, which is 144 yards longer than broad, requires 1076 yards of fence to surround it. Find its length and breadth.

Ans. 341 and 197 yards.

28. The locomotives, Pegasus and Hecla, running in opposite directions on parallel lines of railway, pass each other with the velocity of 80 miles an hour; and, were they running in the same direction, Pegasus would pass Hecla with the velocity of 4 miles an hour. Required the rate of each.

Ans. P. 42, H. 38 miles an hour.

29. If a boatman can row 5 miles an hour with the stream, or $3\frac{1}{3}$ miles an hour against it, what is the hourly velocity of the current?

Ans. $\frac{5}{6}$ mile.

30. A boatman rows 10 miles up a river, and back

again, in 5 hours, the stream flowing uniformly in one direction; and he finds that he can row 5 miles with the stream in the same time that he can row 2 miles against it. Find the velocity of the stream, and the times of going and returning.

Ans. $2\frac{1}{10}$ mi. an hour; $3\frac{4}{7}$ and $1\frac{3}{7}$ hours.

31. The less of two numbers is $7\frac{1}{2}$; what must the greater be, if the difference of the numbers is $\frac{5}{13}$ of their sum?

Ans. $16\frac{7}{8}$.

32. What two numbers are those, the greater of which is to the less as their sum is to 40, and as their difference to 32?

Ans. 324 and 36.

33. A boat's crew row from A to B, 18 miles, down the middle current of a river, and, in returning from B to A, they keep nearer the bank, so as to have the opposing current only $\frac{3}{5}$ as strong as the mid stream. If they row down in $1\frac{1}{2}$ hour, and up in $2\frac{1}{4}$ hours, at what rate could they have returned against the middle current?

Ans. 7 miles an hour.

34. A can do a work in 12 hours, which B can do in 15 hours; find, without recourse to any rule of fractions, the time that A and B together would take to do it.

Ans. 6 hours 40 minutes.

35. A and B together do a work in 12 days, for which A by himself would require 21 days; find, with-

out recourse to any rule of fractions, in what time B alone would do it.

Ans. 28 days.

36. E can perform a work in 21 hours, for which F alone requires 24 hours, and G alone 28 hours; in what time would the three conjointly finish the work?

Ans. 8 hours.

37. F, G, and H, can severally perform a work in $\frac{3}{11}$, $\frac{7}{20}$, and $\frac{5}{14}$, of a week, respectively; what proportion does F's ability bear to the conjoint ability of G and H?

Ans. 35 to 54.

38. M can reap $1\frac{1}{2}$ acre in $2\frac{1}{4}$ hours, N $1\frac{1}{2}$ acre in $2\frac{1}{2}$ hours; in what time will they together reap $10\frac{1}{2}$ acres? And, if the rate of payment be 1s. per acre, what ought each to receive when the work is done?

Ans. $8\frac{1}{38}$ hours; M, 5s. $6\frac{6}{19}$d., &c.

39. W can build a wall in 16 days, R could do it in 23 days; if they work together for a certain time, after which W alone finishes the remainder in 3 days, how long do they work together?

Ans. $7\frac{2}{3}$ days.

40. A party of labourers take 35 days in performing a work, for which another party, consisting of 3 men fewer, would require 40 days. How many men in each party?

Ans. 24 and 21.

41. D can perform a work in 21 hours, and E can do the

same in 22 hours. They work together for 2 hours, at the end of which time E withdraws; D continues working, and in an hour afterwards is joined by F, and when these two have wrought together 3 hours, it is found that the work is only half finished. In what time could D, E, and F, together, do the remaining half?

Ans. $3\frac{45}{62}$ hours.

42. To accomplish a certain work, B would take thrice as long as A, and C would take half as long again as B; and the three agents together would do the work in 9 hours less than B by himself would require. Find the time of each singly.

Ans. A $3\frac{9}{11}$ hrs., B $11\frac{5}{11}$ hrs, C 17 $\frac{2}{11}$ hrs.

43. When wax candles are 2s. 6d. per lb., a composition is invented such that a candle made of it will burn $\frac{2}{3}$ of the time that a wax candle, of the same thickness and $\frac{1}{4}$ as heavy again, would burn. If the two give an equally bright light, what must be charged per lb. for the composition that it may be as cheap as wax?

Ans. 2s. 1d.

44. In what time can 8 men perform a work which 7 boys can do in 12 hours, if 10 men and 2 boys can do as much per hour as 7 men and 10 boys?

Ans. 3 hrs. $56\frac{1}{4}$ min.

45. A certain quantity of calico cost me so much a yard, that by selling it at 9d. I should gain 7s. 8d., but

by selling it at $6\frac{1}{2}$d. I should lose 1s. 11d. Required the quantity and prime cost.

Ans. 46 yds. at 7d.

46. A man intending to give 4d. each to some poor persons, found his purse to contain 14d. too little for the purpose; he gave each person $2\frac{1}{2}$d., and had $5\frac{1}{2}$d. remaining. How many poor persons were there?

Ans. 13.

47. If a rise of interest, from $3\frac{1}{4}$ to $3\frac{1}{2}$ per cent., increases a person's net income (after deducting income tax of 7d. in the pound) by £29, 2s. 6d.; from what principal sum is the income derived?

Ans. £12000.

48. If incomes above £150 pay 7d. in the pound, and those under it pay 5d., what income under £150 is practically equivalent to £151, 5s. $7\frac{1}{2}$d.?

Ans. £149, 19s. $10\frac{1}{2}$d.

49. What must be the market value of 3 per cent. stock, that it may yield $3\frac{1}{3}$ per cent. interest, after deducting income tax of 7d. in the pound?

Ans. $87\frac{3}{8}$.

50. Suppose a person invests £4800 in the 4 per cents., at 80, and at the end of each year invests the dividend which becomes due, in the same stock, at the same price; what will be his dividend at the end of the third year?

Ans. £264, 12s.

51. Given £14, 5s. $8\frac{4}{7}$d., the present worth of £14, 15s.

due one year hence; to find the present worth of £14, 4s. due two years hence.

Ans. £13, 6s. 8d.

52. If by selling goods at a certain price with 6 months' credit, my present gain on the prime cost is 2 per cent.; what term of credit would occasion a present loss of $2\frac{1}{4}$ per cent. on the prime cost, the use of money being worth 5 per cent. per annum?

Ans. $16\frac{16}{23}$ months.

53. Divide £423 into two sums, such that the true discount on one for 5 months, at 4 per cent. per annum, shall equal the present worth of the other for 16 months at 3 per cent. per annum.

Ans. £$415\frac{10}{11}$, and £$7\frac{1}{11}$.

54. An eight gallon cask is full of brandy, and a ten gallon cask is full of water; how much must be transferred from each to the other, that the mixtures may be of equal strength?

Ans. $4\frac{4}{9}$ galls.

55. There is a division of the labour of a certain manufacture between two sets of men, neither of which can do the other's work. The one set consists of 3 men, and the other of 5; and when they work in this proportion, both sets are just fully employed. One man of the first set stays away for a week; by what fraction are the earnings of each of the remaining men thereby diminished, supposing them to work by the piece, and to divide their earnings equally?

Ans. $\frac{5}{21}$.

56. My watch, which was set accurately to-day at 12 o'clock, indicates 10 minutes to 5 at 5 o'clock; what will be the true time when the watch indicates 5 o'clock?

Ans. 10 $\frac{10}{29}$ min. past 5.

57. At what time between 9 and 10 o'clock do the hour and minute hands of a clock point in opposite directions? at what time are they 15 minute spaces apart? and at what time do they point in one direction?

Ans. $16\frac{4}{11}$, $32\frac{8}{11}$, and $49\frac{1}{11}$ min. past 9.

58. At what time between 5 and 6 o'clock are the hour and minute hands of a watch 13 minute spaces apart?

Ans. $13\frac{1}{11}$ and $41\frac{5}{11}$ min. past 5.

59. The hands of a clock being ten minutes apart at 10 o'clock, at what time will they be at the same interval again for the ninth time after 10 o'clock?

Ans. $5\frac{5}{11}$ min. past 3.

60. On leaving home, I observed by my watch that it was between 4 and 5 o'clock, and on returning the same evening, between 7 and 8, I found that the hands had exactly changed places. At what o'clock did I leave home?

Ans. $36\frac{2}{13}$ min. past 4.

61. A hare is 50 leaps before a greyhound, and takes 4 leaps to his 3, but 2 of the greyhound's leaps are as much as 3 of the hare's; how many leaps must the greyhound take to catch the hare?

Ans. 300.

62. Suppose a train to start from Edinburgh for Glas-

gow, the distance being 46 miles, at the rate of 21 miles per hour, and that, 25 minutes afterwards, a locomotive is sent after it, with instructions to overtake it on reaching the terminus; at what rate must the locomotive travel to effect this purpose?

Ans. $25\frac{1}{14}\frac{3}{9}\frac{9}{9}$ miles an hour.

63. There are two places 25 miles apart, from which, respectively, A and B set out, and travel towards each other, A going $3\frac{1}{4}$, B $2\frac{1}{2}$ miles an hour. B starts an hour and a-half later than A. How far will A have travelled when they meet?

Ans. $16\frac{1}{4}$ miles.

64. E and C set out, at the same time, from Edinburgh and Carlisle, respectively, and each travels towards the original station of the other, E's rate being $\frac{3}{4}$ of C's. They meet in 8 hrs. 40 min. How many hours does each person take to the whole journey?

Ans. E $20\frac{2}{9}$, C $15\frac{1}{6}$ hrs.

65. A, B, and C, travel from the same place, in one direction, at the respective rates of $2\frac{3}{4}$, 3, and $4\frac{1}{2}$ miles per hour, and B starts 2 hours after A; how long after B must C start, that they may both overtake A at the same time?

Ans. $7\frac{1}{3}$ hrs.

66. A debt of £760, 10s. 7d., due at present, is to be converted into three equal sums, payable in 4, 7, and 9 months, respectively; what should each payment be?

Ans. £260, 10s. 8·05d.

67. A lb. of tea and 3 lbs. of sugar together cost 6s.;

but, if sugar were to rise 50 per cent., and tea 10 per cent., the same quantities would together cost 7s. What is the price of each?

Ans. Tea 5s., Sugar 4d. a lb.

68. Last week I bought 22 lbs. of beef and 20 lbs. of mutton for 23s. $11\frac{1}{2}$d.; this week the same quantities would cost me 24s., mutton having fallen in price $1\frac{1}{2}$ per cent., and beef having risen 2 per cent; what did I pay per lb. for each article?

Ans. Beef $6\frac{1}{4}$d., Mutton $7\frac{1}{2}$d.

69. Suppose that F, G, and H, start together at one point to travel in the same direction in a circular path of $56\frac{1}{4}$ miles; and that their respective rates are $3\frac{1}{3}$, $9\frac{1}{6}$, and $12\frac{1}{12}$, miles per hour; how soon will they all come together again?

Ans. $19\frac{2}{7}$ hrs.

70. From a point in a circular path of 55 miles, three travellers, A, B, C, set out at the same time, A and B going round it in opposite directions, and C going and returning along the diameter, till they all met together again at the starting point. How long must they have travelled, if A's rate was $7\frac{1}{2}$ miles an hour, B's 9, and C's 6, assuming the circumference of a circle to be $3\frac{1}{7}$ times the diameter?

Ans. $256\frac{2}{3}$ hrs.

71. There is a tank into which water flows constantly and uniformly. If, when it has received 525 cubic feet of water, 11 equal discharging pipes being set open, and running uniformly, would empty it in

48 minutes, and 6 of these pipes could do the same in 98 minutes; at what rate per minute does the water flow into the tank?

Ans. $1\frac{19}{56}$ cub. ft.

72. If 22 oxen eat $7\frac{1}{2}$ acres of grass in 27 days, and 31 oxen eat $8\frac{3}{4}$ acres of similar pasture in 21 days, the grass growing uniformly; how many oxen will exhaust the pasture of $6\frac{1}{4}$ acres in 24 days?

Ans. 20.

73. If 18 oxen in 5 weeks eat 6 acres of grass, and 45 oxen eat 21 acres in 9 weeks, the grass growing uniformly; how many weeks' food for an ox is there on each acre before they begin to feed, and how many weeks' food grows on an acre per week?

Ans. $9\frac{9}{14}$ wks. food at first; $1\frac{1}{14}$ wks. food per wk.

74. What is the extent of a field of grass, which 26 oxen can consume in 10 weeks, if they could eat 7 acres of it in 4 weeks, or 8 acres of it in 5 weeks, the grass growing uniformly?

Ans. 11 ac. 32 po.

75. If 29 oxen eat $6\frac{3}{4}$ acres of grass in 3 weeks, and 23 oxen eat 9 ac. 1 ro. 32 po. of similar pasture in 7 weeks, the grass growing uniformly; in how many weeks will 28 oxen exhaust the pasture of 7 ac. 32 po.?

Ans. $3\frac{3}{7}$ wks.

76. A farmer, who threshes 1000 quarters of wheat annually, saves a shilling on every quarter by using a threshing machine. In erecting this machine he

sunk £200; how much of this sum will his savings have paid back to him at the end of five years, allowing 5 per cent. interest on the sum which remains unpaid at the end of each year, and £5 annually for repairs?

Ans. £193, 7s. 11·3d.

77. S and T are two schools, jointly consisting of 184 pupils. S contains 15 more girls than boys, and T 9 more boys than girls. Also, in S, the number of boys is to that of girls, as, in T, the number of girls is to that of boys. How many boys are in each school?

Ans. S 50, T 39.

78. If barley weighs $51\frac{1}{2}$ lb. per bushel, and when made into malt is increased 3 per cent. in bulk, but loses 16 per cent. in weight, what will be the bulk of 105 lbs. of malt?

Ans. $2\frac{1}{2}$ bu.

79. A, B, C, D, together, do a piece of work, which A, by himself, would take $1\frac{3}{4}$ hr. less than B to do. A and B together would do it in $\frac{7}{8}$ of the time C and D would take, A and C in $\frac{8}{9}$ of the time of B and D, or A and D in $\frac{127}{128}$ of the time of B and C. In what time would each, singly, perform the work?

Ans. A 14, B $15\frac{3}{4}$, C 16, D 18 hrs.

80. A and B owed me together £1000, but I was obliged to accept a composition of 6s. in the pound from A, and 4s. 8d. in the pound from B, and I thereby lost £725. How much did each person owe me?

Ans. A £625, B £375.

PART THIRD.

ANSWERS TO QUESTIONS ON METHOD.

1. Intelligence, accuracy, neatness, and expertness, in the ordinary calculations of business. A habit of reasoning strictly in all matters of discussion, and of expressing thoughts clearly and in good order. A fitness for entering upon the study of algebra, geometry, and other parts of mathematics.
2. The successful teaching of Arithmetic depends partly on the ability to interest a pupil's mind by such lessons and questions as are adapted to his power of judgment, and to gratify his love of being able to understand new forms of truth,—partly on a correct and familiar knowledge of principles,—partly on the expert accuracy which is acquired by careful practice.
3. The obvious connexion of Arithmetic with so many objects familiar to children, suggests early attention to it as natural. The idea of number, too, is natural to the mind; and the demonstrative character of Arithmetic possesses a simplicity suited to the comprehension of a child. Therefore Arithmetic may properly be taught to children at their first admission to school

While children are in the lowest class, they may be taught to count by means of sensible objects, such as the sliding balls of the arithmeticon. When in the lowest class but one, they may learn to perform easy additions, subtractions, &c., with the assistance of the ball-frame, and also by means of properly graduated questions in the computation of familiar things.*

For a class learning the first principles of notation, the earliest lesson should make them acquainted with the nine significant figures, and their simple or general values; and they should be informed that, as it would be impracticable to employ a distinct figure for every consecutive number, it has been thought convenient, when we have got as high as *nine*, to use the figures 1, 2, 3, &c., over again to denote tens, indicating that ten-fold value by writing the figure one place to the left of units. For illustration of this, we may compare half-sovereigns and shillings, or we may employ large and small seeds, pebbles, &c.

It may easily be shewn how all numbers from *ten* to *ninety-nine* may be expressed with two figures; and the pupil should have acquired an intelligent facility in the notation of tens and units, before his attention is directed to that of hundreds and thousands. It is inexpedient to go beyond thousands, till there has been considerable

* See Nelsons' "First Lessons in Arithmetic."

practice in the addition of numbers whose sum is under 10,000.

Ordinary examination by means of the annexed example might be varied as follows:—

Read the figures of the fourth or right-hand column. (Eight units, seven units, &c.) Read the third and fourth together as units. (Thirty-eight units, seven units, &c.) Read the third as units. (Thirty units, no units, &c.) Read the third as tens. (Three tens, no tens, &c.) Read the second and third together as tens. (Twenty-three tens, sixty tens, &c.) . . . Read the largest of the numbers. What would it become if deprived of its thousands?—of its hundreds? What, if the figures denoting tens and hundreds should exchange places?	238 5607 365 14 2065 952

4. The children of a class, having each a copy of the same arithmetical treatise, may be called upon in turn to read the numbers of an addition or subtraction sum, as practice in numeration, or to perform successive portions of the addition or subtraction. The same may be done in regard to multiplication and division, if the book contains worked examples of these. Occasionally, part of the descriptive or demonstrative text may be read, and questions put by the teacher to ascertain how far the meaning is understood.

5. Simultaneous instruction economizes time, but, as the children in an arithmetical class will generally

be found unequal or dissimilar in aptitude for comprehending the principles and processes that are thus taught, individual instruction should be resorted to, before and after the simultaneous is given. The previous individual examination will make the teacher aware of the capacities of the children, and the different kinds of association by which they reason; and he will thus be enabled to adapt the simultaneous lesson more effectively. Before individual instruction is begun, those members of the class from whom the teacher's attention is to be withdrawn, may have slate-work prescribed to them, as preparatory to the individual lesson, or may be placed under the superintendence of a pupil teacher or monitor.

6. After a testing exercise has been prescribed, the black board may be used, to direct attention to the merit of any pupil's performance; to point out sources of error or of liability to mistake; to exhibit any desirable improvement, as regards neatness, brevity, or clearness; and to explain other methods of obtaining the required result.

7. Expediency will be promoted, by using the black board in presenting orderly processes for imitation, and by allowing the pupils to dictate simultaneously, or in turn, the successive steps of the operation performed on the board, thus engaging their attention and understanding.

The teacher should avoid, as inexpedient, a common tendency to be too suggestive of the

answer, in calling on a pupil to take part in the work. He should also guard against the temptation to display his own expertness beyond the rate with which pupils can keep pace. Giving out sums on the board should be much less frequent than by oral dictation,—the latter method involving a better test of the pupil's knowledge of figures, orthography, &c.

8. Arithmetic generally requires a separate classification, because children whose capacities fit them to be associated in one class for reading, history, geography, &c., are generally found very dissimilar in their powers and methods of arithmetical reasoning and calculation.

The requirement of a separate classification may be met, by making Arithmetic the last of the school lessons, morning and afternoon. By this means, there will be only one alteration of class arrangement during school time.

Some teachers, however, make the arithmetical instruction of their upper classes entirely individual, thereby dispensing with the re-arrangement of classes.

9. For the prevention of copying,—if the class is standing, make every alternate pupil stand forward from the rest; if the class is seated, make every alternate pupil stand. When a sum has been dictated to the children thus selected, let them resume their places, and then let a different sum, of the same kind, be given out to the rest.

The superintendence of a pupil teacher may assist in the prevention of copying.

10. In communicating "short methods," a teacher should provide and ascertain that the pupil understands thoroughly the principles involved; and he should never introduce such methods till the ordinary processes are well understood and familiar.

11. Let the first, or right-hand column, be regarded as containing units of the lowest or first rank, each being worth *one;* the second column as containing units of the second rank, each worth *ten;* and so forth. Then, remembering that tens of the first rank are units of the second, tens of the second are units of the third, &c., we may add each column as one of units, dividing its sum into tens and units, and either carrying the tens, or reserving them in a lower line, as in the example, for subsequent addition.

369
5073
7409
874
2505
1122
13725

Addition, as usually worked, may be verified by the above method, beginning, however, with the left-hand column, and adding downwards, to prevent recurrence of any erroneous addition. Or we may cut off the lowest line, find the sum of the others, and then add the lowest line to the sum thus found.

The ordinary method of cutting off the top line does not sufficiently guard against a repetition of error.

12. The natural method begins with subtracting 9 units from 17 units,

927=8 hund.	11 tens	17 units
749=7	4	9
178=1 hund.	7 tens	8 units

one of the 2 tens of the minuend being transferred to the place of units; then, 4 tens are subtracted from 11 tens, one of the 9 hundreds being transferred to the place of tens. This method is not so convenient as the artificial one commonly employed, which is founded on the principle, that the difference of two numbers is the same as the difference of two other numbers equally greater than the two former. This principle may be illustrated by familiar examples, thus:—

John's age is 16 years, and in three years more will be 19
Henry's age is 12 years, and in three years more will be 15

4 years, difference, continues to be 4

While the expedient employed in the natural method makes no alteration in the value of the given numbers, the artificial method increases them; thus, the 7 units and 2 tens of the minuend, not allowing the subtraction of the figures under them, are increased, respectively, by 10 units and 10 tens; and this increase being equal to 1 ten and 1 hundred, can

9 hund.	12 tens	17 units
8	5	9
1	7	8

be applied to the lower number in the places of tens and hundreds, as shewn in the example. The difference of the two numbers after they are thus equally increased, will, according to the principle above stated, be the same as that of the given numbers.

The artifice of borrowing ten may be compared with the fact that other numbers may be found which, when added to the given numbers, will make each figure of the subtrahend of less value than the figure above it. Thus, by adding 62 to each of the given numbers, each figure of the upper line will allow the subtraction of the figure below it. The trouble, however, of ascertaining different numbers to suit different examples in this way, shews at once that the uniform addition of 10 is to be preferred; for the addition of *ten* to any figure of the upper line being compensated by an equivalent addition of *one* to the figure of next higher rank in the lower line, will always leave the under figure small enough for subtraction.

$$\begin{array}{r} 927 + 62 = 989 \\ 749 + 62 = 811 \\ \hline 178 + \ 0 = 178 \end{array}$$

13.

809035d.	8,10,19,10,13,5,d.
509962	6, 1,10,10, 6,2,
299073d.	2, 9, 9, 0, 7,3,d.

Adding *ten* to each figure of the upper line that

does not allow immediate subtraction, and *one* to each figure of next higher rank in the lower line, makes the tens, hundreds, &c., assume the altered values shewn in the example.

The usual method of verification is, to add the two lower lines, and ascertain the correctness of the work by the agreement of the sum with the upper line; for the part taken away added to that which is left, must make up the whole.

Both in addition and in subtraction, the given quantities must be used in one denomination. We cannot say that 2 beans and 3 peas make 5 beans or 5 peas; but if we use one common denomination, as 2 seeds and 3 seeds, the sum is 5 seeds. In like manner, we cannot take from a quantity what it does not contain. We cannot take 3 nuts from a basket of apples. Accordingly, in the given example, the minuend being *pence*, the subtrahend also is *pence*, and the remainder is *pence*.

14. The object of Multiplication is to find the amount of a quantity taken a certain number of times. The aggregate parts being all the same, as in the annexed example, we can perform the addition by saying, 3 times 8 are 24, instead of 8 and 8 are 16, and 8 are 24, &c. Therefore we need not write the same number repeatedly; but having written it once, and placing 3, the number of times, under it, we can proceed by the usual mode of multiplication.

278
278
278
834

15. If the product of 72 times a quantity were found by addition, the aggregate parts, all equal, would be 72 in number, and the whole might be divided into portions of eight lines each, the sum of each portion being 8 times the single quantity. Now, in 72 lines there would be 9 portions, each consisting of 8 lines; therefore the whole amount would be 9 times the amount of 8 lines.

16. If we have 5 shilling pieces in a row, 5 pieces more in a second, and 5 more in a third row, as in the margin, there are 3 rows, each containing 5 shillings, or 5 rows, each containing 3 shillings: therefore 3 times 5 shillings = 5 times 3 shillings.

o o o o o
o o o o o
o o o o o

17. To multiply units by 10 is to make them tenfold as great in value, or to entitle them, as it were, to be removed one place to the left. The same may be said respecting tens, hundreds, &c.: therefore to multiply any number by 10 is the same as to remove each of its figures one place to the left; which is done by annexing one cipher on the right, to occupy the place of units. That multiplying by 100 requires merely to annex two ciphers may be similarly explained.

The mode of multiplying by 30, 400, &c., is obviously deducible from the preceding explanation; for 30 is 10 times 3, and the product by 3 is multiplied by 10 when a cipher is annexed to that product. Similarly, 400 is 100 times 4, and

the product by 4 is made 100 times as great by annexing two ciphers.

18. The multiplication proposed is to find the amount of 47302 taken 7052 times; which may be done by adding together 2 times, 50 times, and 7000 times the multiplicand.

47302
7052
94604
2365100
331114000
333573704

The first partial product is formed thus: 2 times 2 units are 4 units, 2 times no tens make no tens, 2 times 3 hundreds make 6 hundreds, 2 times 7 thousands make 14 thousands, or 4 thousands and 1 of the next higher rank, which being carried to the next result, 8, gives 9 tens of thousands.

The second partial product is that by 50. Now, to multiply by 50, we may first take 5 times the given number, and then 10 times the result; but 10 times the result will simply remove each figure of that result one place to the left, letting a cipher occupy the place of units. We therefore multiply by five, as if it denoted units, but make the figures of the result advance one place farther to the left, by pre-occupying the place of units with the cipher.

The third partial product is 7000 times the given number; to find which, we multiply by 7 and by 1000, letting three ciphers on the right denote the thousand-fold product of 7 times.

The usual omission of the ciphers that arise from the local values of the digits in the multiplier

is expedient in practice, but they should be retained in demonstration; the partial products then have the proper form of an addition sum.

19. Add the digits of the multiplicand from left to right, rejecting nine whenever the sum equals or exceeds nine. Thus: 4 and 7 are 11, 2 and 3 are 5 and 2 are 7, the final remainder, to be written a little to the right. Next add the digits of the multiplier similarly, and write the final remainder 5. Lastly, add in the same manner the digits of the product, and write the final remainder 8. The test consists in applying to the product of 5 times 7 the same process of adding its digits 3 and 5, and rejecting nine if the sum admits. The remainder is 8, the same as that resulting from the product of the given numbers; hence the work is presumed to have been correctly performed.

47302...7
7052...5
333573704...8

In proceeding to demonstrate the above method, let the multiplicand and multiplier be each divided by 9; they will be thus resolved into the equivalent expressions—

5255 nines + 7 units.
783 nines + 5 units.

Now, these expressions manifestly indicate that their product will be an exact number of nines + 35 units; and therefore their product divided by

nine will have no remainder but what arises from the division of 35 by nine.

In like manner, it may be shewn, that if any common divisor is applied to a multiplicand and multiplier, and the two remainders are multiplied together, the product of these remainders and that of the given factors will yield one common remainder to the common divisor.

Division, therefore, might serve in this way to verify multiplication; but for learners it would be inappropriate thus to anticipate a higher rule. The number 9, however, as a divisor, possesses a property from which an easily available mode of verification may be derived. The property is this (and 3 is the only other number having the same property): that dividing a number by 9 yields the same remainder as dividing the sum of its digits by 9. Thus, 3758 divided by 9 leaves 5; also $(3 + 7 + 5 + 8)$ divided by 9 leaves 5: so that to find the remainder we need employ no higher rule than subtraction, casting out, or subtracting, nine as often as the summing of the digits allows.

That this property of 9 is general, may be thus shewn:—

10, or any power of 10, when divided by 9, leaves 1; therefore 3 times, 4 times, &c., any power of 10 leaves 3, 4, &c.

Example: Since 1000 is an exact No. of nines + 1, therefore 5000 is 5 times that No. of nines + 5

Consequently, to find the remainder arising from the division of 3758 by 9, we have

3000,	an exact No.	of nines	+ 3
700,	do.	do.	+ 7
50,	do.	do.	+ 5
8,	do.	do.	+ 8

Therefore, 3758, an exact No. of nines + 23,
Or, an exact No. of nines + 5.

"Casting out the nines" is only a presumptive test of accuracy, because the digits of the product might be transposed or otherwise altered, without affecting their sum.

As another mode of verifying, we might make the multiplicand and multiplier exchange places, and so obtain partial products different from those previously found.

20. A multiplier always denotes so many times the multiplicand, and therefore must always necessarily be an abstract number.

If 1 man for 1 day receives 3s.,
6 men for 1 day get 6 times 3s.
That is, not 3s. × 6 men, but
3s. repeated as many times as there are men.
∴ 6 men for 4 days get 24 times 3s.

21. Frequent practice in the mental addition of equal aggregate parts; committing to memory the series of products in reverse order, as 7 times 12, 7 times 11, &c.; frequent dictation of sums in multiplica-

tion that require the products most difficult to recollect, such as 7 times 8; 6, 7, and 8 times 9; 8, 9, and 11 times 12; and the inversions of these.

22. The divisor is not contained in 3, but is contained in 30; therefore the highest place in the quotient will be thousands. 9 days are contained in 30 thousand days 3 thousand times, leaving 3 thousand, or 30 hundred days, which, with the given 8 hundred, make 38 hundred; 9 days are contained in 38 hundred 4 hundred times, leaving 2 hundred, or 20 tens, which, with the 2 tens given, make 22 tens of days; and so forth.

9 da.) 30827 da.

3425$\frac{2}{9}$ times.

The final remainder, 2 days, is also required to be divided by 9 days; and it is perhaps difficult to explain such division to an elementary class, further than by saying, that 9 days are contained in 2 days as often as 9 is contained in 2, viz., the 9th part of 2 times; for 2 times would amount to 18, and 2 is only the 9th part of 18. The representation of the 9th part of 2 times by the form $\frac{2}{9}$ requires description merely, not demonstration; it is a form chosen to denote the quotient of 2 divided by 9. It will be easily seen what is meant by 2 being divided by 9, when the pupil is reminded how 2 flocks of sheep might be divided into 9 equal parts, if each flock contained 18 sheep.

In proceeding to divide by 59, we find that the divisor is not contained in 3, or in 30, but is contained in 308; therefore, the highest value in the quotient will be hundreds. Now, because 59 is contained 5 times in 308, it is contained 5 hundred times in 308 hundred, and by multiplication and subtraction we find that there are still 1327 days to be divided by 59 days; accordingly, as 59 is contained 2 times in 132, it is contained 2 tens of times in 132 tens, &c.

days.	days.	times.
59)	30827	(500
	29500	20
		2
	1327	—
	1180	$522\frac{29}{59}$
	147	
	118	
	29 da.	

23. By the process of Division we find, either, how many times one quantity is contained in another of the same kind; or, what quantity is contained a certain number of times in another of the same kind.

In dividing 36 ounces by 4 ounces, we find that 4 ounces are contained in 36 ounces 9 *times*. In dividing 36 ounces by 4, we find that the 4th part of 36 ounces, or the quantity contained 4 times in 36 ounces, is 9 *ounces*.

In the latter of these examples, the number 4, though occupying the place of a divisor, is strictly the quotient, as denoting *how often* the required quantity, 9 ounces, is contained in 36 ounces.

24. The quotient, that is, the number which denotes merely the number of times, is always an abstract number.

The process of Division may be verified by multiplying the divisor by the quotient, in expectation of thus reproducing the dividend. A remainder, if any, must be added to the product.

25. In the Italian method of performing Long Division, the successive remainders are found by mental subtraction, the subtrahend not being set down; thus:—

Italian Method.	Common Method.
269)1140158(4238	269)1140158(4238
641	1076
1035	———
2288	641
136 rem.	538
	———
	1035
	807
	———
	2288
	2152
	———
	136

As 269 is contained 4 times in 1140, we say 4 times 9 are 36, and 4 (set down 4) are 40; carry 4; 4 times 6 are 24, and 4 are 28, and 6 (set

down 6) are 34; carry 3; 4 times 2 are 8, and 3 are 11 (no remainder). To the figures 64, thus written down, annex the next figure of the dividend, 1, and repeat the procedure.

26. In dividing 3760 by 6 we resolve the given dividend into 626 sixes + 4 units; dividing again by 9, we have resolved the given dividend into 69 fifty-fours + 5 sixes + 4 units; so that the remainder is 5 sixes + 4, or 5 times the first divisor + the first remainder.

```
6)3760
  ----
9)626 sixes   + 4 units.
  ----
69 fifty-fours + 5 sixes.
                  ---
                  34 rem.
```

27. Assuming it as proven (see No. 16) that 3×6 is $= 6 \times 3$, we have $48 \times 6 = (16 \times 3) \times 6 = 16 \times (6 \times 3)$ = a third of 48, × three times 6.

28. $48 \div 6 = 16$ threes $\div 2$ threes; and 2 threes are evidently contained in 16 threes just as often as 2 units in 16 units;

$$\therefore 48 \div 6 = \tfrac{48}{3} \div \tfrac{6}{3}.$$

Universally, a quotient is not altered by multiplying or dividing the dividend and divisor by any one number; because the number thus used to increase or to diminish the terms, has merely the effect of a common change of denomination. Thus: $84 \div 24 = 7$ dozen $\div 2$ dozen $= 7 \div 2 = \frac{84}{12} \div \frac{24}{12}$. Thus also $26 \div 14 = 26$ sevens $\div 14$ sevens $= 182 \div 98$.

29. To multiply by 12 and then divide by 18, is to take the 18th part of 12 times, which is equal to the 18th part of (1 + 1 + 1 + &c.) times = 12 times the 18th part of once. Therefore,

The 18th of 12 times a No. = 12 times the 18th of that No.

To use the multiplier first is generally preferable, because in the other way a fraction may arise in the middle of the work.

To multiply by 12, and then divide by 18, is to take the 18th part of 12 times or (12 ÷ 18) times, and 12 ÷ 18 = 2 sixes ÷ 3 sixes = 2 ÷ 3, or the 3rd part of 2 times.

30. £ *s.* *d.* are the initial letters of the Latin words *Libra*, *Solidus*, and *Denarius;* which respectively mean Pound, Shilling, and Penny. *Lb.* is a different abbreviation of *Libra*, a pound.

Cwt. is the initial letter of the Latin *Centum*, hundred, prefixed to an abbreviation of the word *weight*, and refers to 112 lbs. viz., 100 with extra allowance of 12.

Dwt. is the initial of *Denarius*, a penny, prefixed to *wt.*, and refers to the weight of the silver penny, which was a 240th of the Saxon pound.

31. As £1 is worth 20s., therefore £53 is worth 53 times 20s., or 20s. × 53 = 1060 shill.; whereas £53 × 20 denotes 20 times £53, or £1060.

32. *First*, As 1 cwt. is equal to 4 qrs., therefore, 5 cwt. = 4 qrs. × 5, or 20 qrs.; hence, 5 cwt. 3 qrs. = 23 qrs.

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Again, 1 qr. = 28 lbs., and 23 qrs. = 28 lbs. × 23, or 644 lbs.; hence, 5 cwt. 3 qrs. 9 lbs. = 653 lbs.

Secondly, Every 4 farthings in 4694 farthings makes 1 penny; so that 4694 f. ÷ 4 f. = 1173½d.

Again, every 12 pence in 1173½ pence makes 1 shilling; so that 1173½d. ÷ 12d. = 97s. 9½d.

And again, every 20 shillings in 97 shillings makes £1; so that 97s. ÷ 20s. = £4, 17s.

Therefore, 4694 farthings = £4, 17s. 9½d.

The work may be verified by reversing the questions, and reducing 653 lbs. to cwt., &c., and £4, 17s. 9½d. to farthings.

33. *First*, Take one of the half-crowns, and ask the change for it in pence; half-a-crown is worth 2s. 6d., or 2 times 12d. and 6d. more; that is, 24d. and 6d., or 30d., for a single half-crown. Accordingly, for 39 half-crowns, the change would amount to 39 times 30 pence, or 1170 pence.

Secondly, 1 yd. = 3 ft. and

47 yds. = 3 ft. × 47 = 141 ft.
½ yd. = ½ of 3 ft. = 1½ ft.

47½ yds. = 142½ ft.

34. There is not an exact number of yards in an ell, or of ells in a yard; but the ell is = 5 quarters of a yard; hence, for every 5 quarters in the given quantity there will be so many ells; and therefore reduce the given quantity to quarters:—

1 yd. = 4 qrs., hence, 136 yds. = 4 qrs. × 136 = 544 qrs.; then, 544 qrs. ÷ 5 qrs. = 108 ells 4 qrs.

To reduce 117524 square inches to the higher denominations, the following method may be sufficiently explanatory:—

144)117524

9)816 ft. 20 in.

30¼) 90 yds. 6 ft.

121)360 quarter yds.

2 po. 118 quarter yds. = 2 po. 29½ yds.
29½ yds. = 29 yds. 4½ ft. = 29 yds. 4 ft. 72 in.
to which add 2 po. 0 yds. 6 ft. 20 in.

2 po. 30 yds. 1 ft. 92 in.

35. Compound Addition and Simple have this common feature, that, as in the latter we have columns of units, tens, hundreds, &c., or successively increasing values, so in the former we have columns of farthings, pence, &c., ounces, pounds, &c., and other successively increasing values. Also, in Simple Addition, we divide the sum of an inferior column by *ten*, setting down the remainder, and carrying the quotient to the next higher column;

and similarly, in Compound Addition, we divide the sum of an inferior column by the quantity equivalent to one of the next higher column, setting down the remainder, and carrying the quotient. Such is the extent of the resemblance. In Simple Addition the scale of increase, and the converting divisors are uniform; in the Compound Rule they are not so.

36. Beginning at the place of farthings, we see that 2 farthings do not suit for subtraction from 1 farthing; therefore, according to the expedient of equally increasing the minuend and subtrahend, add 4 farthings to 1 farthing and carry the equivalent 1 penny to the lower 7; 2 farthings from 5 farthings leave 3 farthings; 8 pence from 7 pence cannot be taken, therefore borrow 12 pence to the upper number of pence, and carry 1 shilling to the lower number of shillings; 8 pence from 19 pence leave 11 pence; 14 shillings from 11 shillings cannot be taken, therefore borrow 20s. and carry £1; &c.

£58	11	$7\frac{1}{4}$
12	13	$7\frac{1}{2}$
£45	17	$11\frac{3}{4}$

An easier procedure may be employed. Beginning again, say, 2 farthings from 4 leave 2, and 1 are 3; 8 pence from 12 leave 4, and 7 are 11; 14 shillings from 20 leave 6, and 11 are 17, &c., thus adding the complements of the subtrahend to the values in the minuend.

In performing the required division, the result can be set down as in Short Division, while

the other parts of the work are in the usual form of Long Division.

78)£239 17 5

£3 1 $6\frac{5}{78}$

234

The 78th part of £239 is £3; 78 times £3 is £234; the remaining £5, with 17s. =117s., the 78th part of which is 1s., and 39s. remain; 39s. 5d. =473d., and so forth. The remaining 5d. divided by 78, gives the 78th of 5d., not amounting to 1 farthing.

5

20

117

78

39

12

473

468

5

37. If we make a mark at the lowest point in the circumference, and then roll the wheel along the road, the wheel will have passed over the space of 11 feet when the mark comes again to the ground, or the wheel in turning round once passes over 11 feet of road. Accordingly, in passing over a mile, or 5280 feet, the wheel will turn round as many times as 11 is contained in 5280; that is, 480 times.

38. If 1 yd. cost 3s. 6d., the cost of 7 yds. is 7 times 3s. 6d., or 24s. 6d.

If 9 yds. cost 31s. 6d., the cost of 1 yd. is the 9th part of 31s. 6d., viz. 3s. 6d.

Hence may be derived questions of the following form:—

If 7 yds. cost 24s. 6d., what cost 9 yds.?

If 7 yds. cost 24s. 6d., how many yards cost 31s. 6d.?

The former of these may be solved by first principles, thus:—

Cost of 7 yds. = 24s. 6d. = 294d.

Do. 1 yd. $= \frac{294}{7}$d.

Do. 9 yds. $= \frac{294 \times 9}{7}$d.

= 378d. = 31s. 6d.

To work such questions by first principles is to employ only the elementary rules, Multiplication, Division, Reduction, &c., and thus to shew that no new rule is necessary for questions in Proportion, whatever expediency the method called Proportion statement may in some instances possess.

39. When one quantity, in relation to another, becomes greater or less in proportion as the other becomes greater or less, the proportion is direct. When one quantity, in relation to another, becomes greater or less in proportion as the other becomes less or greater, the proportion is inverse.

Thus, we propose a question in direct proportion when we say,—If 12 men earn 20s., how many shillings are earned by 9 men? because the sum decreases in the direct ratio of the number of men. But the proportion is inverse when we say,—If 12 men take 20 days to perform a work, how many days would 9 men require? because the time increases in the inverse ratio of the number of workmen.

40. Write down the data of that part of the question of which the details are complete, and which naturally follows the word *If*, and so arrange these data that the quantity of the same kind as the answer shall be on the right hand; then convert the left-hand quantity into a unit, and the right-hand quantity into the corresponding value for that unit; lastly, by multiplication, convert the unit into the quantity for which a corresponding value is sought, and make at the same time an equivalent change on the corresponding value of the unit. This last result is the answer.

Note. The multiplications and divisions, generally, should be reserved in the symbolic form, and not worked out till after the conclusion of the reasoning.

41. 17 yds. : 13 yds. :: 43s.

By multiplying 43s. by 13 we obtain the value of 13 yds. at 43s. *per yard*, viz. 559s.; but as 43s. is not the price of 1 yd., but of 17 yds., the amount ought to be one-seventeenth of 559s., and hence the reason of the division by the first term.

42. If 23 men reaped 15 acres *each*, then 16 men would also reap 15 acres each, and would reap in all 15 $\times$ 16 or 240 acres; but it was 23 men, not 1 man, that reaped 15 acres, therefore the 16 men reap only the 23d part of 240 acres., viz., 10 $\frac{10}{23}$ acres.

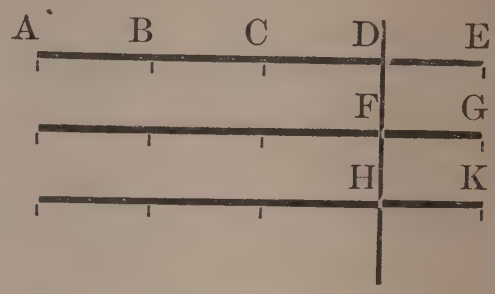
43. A fraction is a quantity less than a unit, but bearing the same denomination as the unit; it is the quotient by a divisor that is greater than the dividend. Thus $\frac{2}{3}$ means the quotient of 2 divided by 3; or it

is 2 third parts of a unit: also £$\frac{4}{5}$ is less than £1, and means the quotient of £4 divided by 5; or it is 4 fifth parts of a pound.

The expressions $\frac{3}{3}$, $\frac{7}{3}$, have the fractional form, but are not proper fractions, because the dividend is not less than the divisor.

An improper fraction is an expression having the fractional form, but the lower number not greater than the upper.

44. Take any whole quantity, say a yard of ribbon, and first let the line A E be drawn to represent the unit, called a yard; let A E be halved at the point C; then A C and C E will each represent half a yard; let these again be halved, and we shall have the whole yard divided into four quarters of a yard. Now, three of these parts make up the line A D, and this line may, if we choose, be regarded as a whole quantity, but not under the denomination of yards; it would be found to measure 27 inches; but if it is to be expressed in relation to A E, and to retain the denomination of yards, then as it contains 3 parts of the 4 into which the yard was divided, it is $\frac{3}{4}$ of a yard.



Again: A D being 3 fourths of 1 yard, may be shewn to be equal to 1 fourth of 3 yards; for the fourth of 3 yards is the fourth of (1 + 1 + 1) yards,

that is, 3 times the fourth of 1 yard. And this equality may be illustrated by taking three equal lines, as above, and shewing that D H in cutting off the three pieces D E, F G, H K, cuts off 3 fourths of a yard = a fourth of 3 yards.

When we take $\frac{3}{4}$ to signify 3 fourth parts of a unit, the lower quantity, denoting the denomination of parts into which the unit is divided, is properly called the denominator; and the upper quantity, which expresses the number of parts taken, is properly called the numerator. The 4 is a reduction divisor, and the 3 a multiplier of the result. But when we take $\frac{3}{4}$ to signify 3 units $\div 4$, the quantities 3 and 4 are simply dividend and divisor.

45. Because $\frac{5}{7}$ signifies $5 \div 7$, and $5 \div 7$ is $= 5$ sixes $\div 7$ sixes, therefore $\frac{5}{7} = \frac{5 \times 6}{7 \times 6} = \frac{30}{42}$.

46. Addition naturally requires that quantities be of like kind that are to form aggregate parts of one whole. As we cannot immediately add together into one sum 10d. and 4s., but require them first to be of one denomination, as 10d. and 48d., so we cannot at once add together £$\frac{1}{24}$ and £$\frac{1}{5}$, because 5ths and 24ths are different denominations of parts. A similar explanation may be given of Subtraction.

47. To multiply $\frac{3}{5}$ by $\frac{4}{7}$ is to multiply $\frac{3}{5}$ by the 7th part of 4. Now, if $\frac{3}{5}$ be taken 4 times, that is, added to itself 4 times, the amount will be $\frac{12}{5}$; but 4 is 7 times as great as the 7th part of 4, therefore $\frac{12}{5}$ or $12 \div 5$ must be further divided by 7, that is, 12 is to be divided by 7 times 5; and hence, $\frac{3 \times 4}{5 \times 7}$ is

the required form for the multiplication of the two fractions.

48. To divide $\frac{3}{5}$ by $\frac{4}{7}$ is to find how often $\frac{4}{7}$ is contained in $\frac{3}{5}$, and therefore we should reduce dividend and divisor to one denomination of parts, viz., 35ths; which is effected by multiplying the numerator of each fraction into the denominator of the other, and gives 7 times 3, thirty-fifths, to be divided by 5 times 4, thirty-fifths, $= \frac{3 \times 7}{5 \times 4}$; which is the form prescribed by the usual rule.

49. As $\frac{2}{3}$ of $\frac{5}{8}$ means that $\frac{5}{8}$ is to be taken $\frac{2}{3}$ of once, the equivalent expression is $\frac{5}{8} \times \frac{2}{3}$. This gives $\frac{10}{24}$, which will become unity if increased by $\frac{14}{24}$; and to find what fraction of $\frac{2}{3}$, that is, what fractional multiplier of $\frac{2}{3}$, will make a product of $\frac{14}{24}$, we must divide $\frac{14}{24}$ by $\frac{2}{3}$, which gives $\frac{7}{8}$.

50. To divide a number by $\frac{1}{7}$ is to find how often it contains the 7th part of unity. Now, it evidently contains that quantity 7 times as often as it contains unity; but 7 times as often as a number contains unity, is just 7 times the number: therefore division by $\frac{1}{7}$ is equivalent to multiplication by 7.

51. Take the fraction $\frac{8}{9}$; its half is evidently $\frac{4}{9}$, which is obtained by dividing the numerator by 2; but half the fraction would as evidently be the result if we made the denomination of parts half as small as 9ths, that is, 18th parts; thus the fraction would become $\frac{8}{18}$, which is $= \frac{4}{9}$. A similar explanation would obviously suit any quantities that might be chosen.

Also, to multiply a numerator is equivalent to dividing a denominator; for $\frac{8}{9}$ is evidently made thrice as great, if we take the given number of 9ths three times, making $\frac{24}{9}$, or if we alter the denominator so as to express parts three times as great as 9ths, that is, 3rds; $\frac{24}{9} = \frac{8}{3}$.

52. When one quantity is said to be to another as 8 to 3, the meaning is that the quotient of the one quantity divided by the other is that of $8 \div 3$, or that the quotient is $\frac{8}{3}$. This relation is called Ratio, and means the relation between two quantities as regards the number of times the one contains the other. Thus, because $40 \div 15$ is $= 8$ fives $\div 3$ fives $= 8 \div 3$, therefore 40 is to 15 as 8 to 3.

53. The expression *equality of ratios* is not strictly accurate. The ratio of 40 to 15, viz., $2\frac{2}{3}$, is not properly said to be equal to that of 8 to 3, viz., $2\frac{2}{3}$; the two are *identical*. It would be more accurate to say that Proportion is the equivalence of two expressions denoting the same ratio.

54. If 12 men take 6 days, fewer men require 8 days; therefore the proportion statement is,—

8 da. : 6 da. :: 12 men : 9 men.

Now, this statement means that $\frac{8}{6}$, expressing the ratio of 8 to 6, is equal to $\frac{12}{9}$, expressing the ratio of 12 to 9; and the equivalence of these expressions may be tested by reducing to a common denominator, thus, $\frac{8}{6} = \frac{12}{9}$, because 8 times 9, fifty-fourths, $=$ 6 times 12, fifty-fourths; that is, the product of the extremes is also that of the means

55. Change into the following equivalent question: How many yards at 66d. should be given for 19 *yards* at 44d.?

44d. for each yard in 19 yds.
amounts to 1d. for each yard in 19×44 yds.
or to 66d. for each yard in $\frac{19 \times 44}{66}$ yds.
$= \frac{19 \times 2}{3} = 12\frac{2}{3}$ yds.

Otherwise. As there will be fewer yards at the higher price, the proportion statement is,—

66d. : 44d. : : 19 yds. : $12\frac{2}{3}$ yds.

56. (*a*) If £74, 11s. purchase an income of £5, 10s., what income will £546, 14s. purchase?

(*b*) If I can get 39 ounces of 48s. wheat, how many ounces of 52s. wheat can I get?

(*c*) If 6 gall. 3 qts. 1 pt. are supplied in 1 minute, in what time will 1000 gall. be supplied?

57. If 6 men, of 4 times ordinary ability, do a work in 10 days of 8 hours each, how many men of 3 times ordinary ability would do the work in 15 days of 6 hours each?

A compound ratio arises from modifying the terms of one simple ratio by multiplying them into the respective terms of another.

58. *First*, write down the data of which the details are complete, making the last expression that which is like the answer. *Secondly*, reduce every expression but the last to a unit, making the last expression receive, successively, the corresponding

alteration for every term thus reduced. *Thirdly*, change all the units into those quantities for which the answer is required, and make the last expression receive, successively, a corresponding alteration for every such change.

(*a*) 125d., when 16 apples weigh 5 lbs., buys $\frac{84 \text{ ap.}}{}$

1d., ,, 1 ap. ,, 1 lb., ,, $\frac{84 \times 5}{125 \times 16}$

20d., ,, 10 ap. ,, 3 lbs., ,, $\frac{84 \times 5 \times 20 \times 10}{125 \times 16 \times 3}$

The result is = 14 apples.

Otherwise, by proportion statement,—

$$\left.\begin{array}{r} 125 : 20\text{d.} \\ 16 : 10 \text{ ap.} \\ 3 : 5 \text{ lbs.} \end{array}\right\} : : 84 \text{ ap.} : 14 \text{ ap.}$$

(*b*) 20 oxen in 7 wks. eat $17\frac{1}{2}$ wks. growth of $\frac{6 \text{ ac.}}{}$

1 ox ,, 1 wk. ,, 1 wks. growth of $\frac{6 \times 17\frac{1}{2}}{20 \times 7}$

35 ox. ,, 12 wks. ,, $22\frac{1}{2}$ wks. growth of $\frac{6 \times 17\frac{1}{2} \times 35 \times 12}{20 \times 7 \times 22\frac{1}{2}}$

The result is = 14 acres.

Otherwise, by proportion statement,—

$$\left.\begin{array}{r} 20 : 35 \text{ ox.} \\ 7 : 12 \text{ wks.} \\ 22\frac{1}{2} : 17\frac{1}{2} \text{ wks.} \end{array}\right\} : : 6 \text{ ac.} : 14 \text{ ac.}$$

59. One quantity is $\frac{3}{8}$ of another when, the latter being divided into 8 equal parts, the former contains 3 such parts; or, when the latter, multiplied by $\frac{3}{8}$, produces the former.

Secondly, B has 1 share, A $\frac{3}{8}$ of a share; A and B together $1\frac{3}{8}$ of a share. The joint amount, therefore, is 11 eighths of a share, as consisting of

A's 3 eighths and B's 8 eighths. Accordingly, A has $\frac{3}{11}$, and B $\frac{8}{11}$ of the joint amount; also, 5 of the 8 parts in B's share, or $\frac{5}{8}$ of B's share, is the excess of B above A.

60. The annexed process, by which 9 is ascertained to be the greatest common measure of 27 and 72, may be easily demonstrated by the aid of the two following principles:—

```
27)72(2
   54
   —
18)27(1
   18
   —
 9)18(2
   18
   —
```

1st. Whatever measures a quantity measures any multiple of that quantity. If 6 measures 18, it must also measure any number of times 18; that is, 18+18+18+&c.

2nd. Whatever is a common measure of two quantities, measures their sum and difference. If 6 measures 24 and 18, it measures also 24+18 and 24—18; for the quotient of 24 by 6 is a whole number, and that of 18 by 6 is also a whole number; therefore, those of 24+18 and 24—18 are, respectively, the sum and the difference of two whole numbers.

Now, in seeking the greatest common measure of 27 and 72, the first step is to try whether 27 be that g. c. m., for, at all events, there cannot be a greater than 27. On trial, then, there is found a remainder of 18, so that 27 is not the common measure. The next step similarly tries to find the

greatest common measure of 18 and 27; for it may be proved that the g. c. m. of 27 and 72 is also that of 18 and 27. The proof is as follows:—

1st. The g. c. m. of 27 and 72 measures twice 27, or 54, and also measures 54 and 72, and therefore measures 72 — 54, or 18, and therefore measures 27 and 18.

2ndly. The g. c. m. of 27 and 18 measures twice 27, or 54, and also measures 54 and 18, and therefore measures 54 + 18, or 72, and therefore measures 27 and 72.

Accordingly, since the g. c. m. of 27 and 72 measures 27 and 18, and since also the g. c m. of 27 and 18 measures 27 and 72, it follows that the g. c. m. of 27 and 72 is neither more nor less than the g. c. m. of 27 and 18.

In continuation of the process, we might similarly shew, that the g. c. m. of 18 and 27 is also that of 18 and the remaining 9, which is immediately found to be 9.

61. Suppose it is required to find the least common multiple of 12 and 16. First, we have 12×16, or 192, a common multiple of 12 and 16; but if we divide 12×16 by any common measure of 12 and 16, we shall evidently have an exact number of twelves, and also an exact number of sixteens. If, for instance, we divide by the common measure 2, the result is 6×16 or 12×8; accordingly, we shall obtain the lowest whole number of twelves and sixteens, when we have divided 12×16 by

the g. c. m. of 12 and 16, that is, by 4, which gives 3 sixteens or 4 twelves, viz. 48.

62. To subtract $25\frac{4}{9}$ from $32\frac{3}{9}$, add unity $= \frac{9}{9}$ to each expression, making the first $= 26\frac{4}{9}$, and the second $= 32\ \frac{12}{9}$; then, as the numbers have been equally increased, the difference is not affected, and is found to be $6\frac{8}{9}$.

63. $\frac{3}{4}$ of a shill. means $\frac{1}{4}$ of a shill. taken 3 times; and $\frac{1}{4}$ of a shill. is $\frac{1}{4}$ of 12d. = 3d.; therefore, $\frac{3}{4}$ of a shill., or $\frac{3}{4}$ of 12d., = 9d.

Then, $\frac{1}{6}$ of half-a-crown should be also expressed in pence, that we may add it to 9d.; and as half-a-crown is equal to 30 pence, $\frac{1}{6}$ of half-a-crown is $\frac{1}{6}$ of 30 pence, = 5d.

Lastly, 9d. and 5d. make 14d., or 1s. 2d.

64. An English ell is 5 quarters of a yard, or $\frac{5}{4}$ yard; therefore, $\frac{5}{7}$ of an ell is $\frac{5}{7}$ of $\frac{5}{4}$ yard; hence, $\frac{5}{4}$ yd. $\times \frac{5}{7} = \frac{25}{28}$ of a yard.

65. When two numbers are in the ratio of 7 to 9, the less is $\frac{7}{9}$ of the greater. Then, the sum of the two is $1\frac{7}{9}$ of the greater, and the difference is $\frac{2}{9}$ of the greater; therefore, the sum being to the difference as 16 ninths to 2 ninths, the difference is $\frac{1}{8}$ of the sum.

66. Decimal fractions are formed by subdividing unity into tenths, hundredths, and so forth, that the decimal scale of numeration may be applied to all values less than unity, as well as to all whole numbers of units.

Vulgar, or common, fractions, are those whose

denominators are not restricted to the scale of decimal subdivision, but exemplify every variety of parts.

In converting $\frac{5}{8}$ to the decimal form, we take 5 units = 5·000, that the 8th part of 5, which is the meaning of $\frac{5}{8}$, may turn out decimally. Thus, the 8th part of 5 gives 0 units; therefore, reduce 5 units to tenths, viz., 50 tenths, the 8th part of which is 6 tenths, with a remainder of 2 tenths; this remainder, reduced to hundredths, is 20 hundredths, the 8th part of which is 2 hundredths, with a remainder of 4 hundredths; this remainder, reduced to thousandths, is 40 thousandths, the 8th part of which is 5 thousandths.

8)5·000
———
·625

67. In proceeding to teach the notation of decimal fractions, it may be expedient to choose some whole number, with a fractional continuation of tenths, hundredths, &c., such as 2634·675, and to represent its value, in the first instance, thus,—

$$2000 + 600 + 30 + 4 + \tfrac{6}{10} + \tfrac{7}{100} + \tfrac{5}{1000}$$

Begin at the left hand, and compare the local values, successively, as thousands, hundreds, tens, *ones*, tenths, hundredths, thousandths, arising from successive divisions by ten. Then, it can easily be explained that these local values are sufficiently distinguishable when the number is written 2634·675, the point on the right of units' place marking the termination of the integral values, and the commencement of the fractional.

Further: as 2634 may be read collectively as 2634 *units*, so may ·675 be read collectively as 675 *thousandths*, and so also may the whole expression be read collectively as 2634675 *thousandth parts* of unity.

The scale of notation, then, being so constructed that the value of any place which a figure occupies is ten times the value of the next place on the right, it may easily be shewn that fractional quantities in the decimal form may be added, subtracted, &c., in the same way as whole numbers.*

This simplification of fractional arithmetic enables us to make money calculations very easily, when the £ is divided decimally into florins, cents, and mils; for florins being tenths of a £, cents being hundredths of a £, and mils thousandths, the representation of £5, 3 flors., 2 cents, 8 mils, would be simply £5·328.

68. As £1 equals 100 cents, therefore, £·475 = 100 cents × ·475 = 47·5 cents = 47 cents and $\frac{5}{10}$ of a cent = 47 cents 5 mils.

Similarly, as £1 equals 1000 mils, therefore £·475 = 1000 mils × ·475 = 475 mils.

The expression £·57$\frac{1}{4}$ means 5 tenths + 7$\frac{1}{4}$ hundredths, of a £. Now, 7$\frac{1}{4}$, or 7·25 hundredths = 7 hundredths + 2 tenths of a hundredth + &c.; so that the given expression altered to the purely decimal form is £·5725.

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* See also the Answer to Question 95.

69. Units, Tenths, Hundths., Thousths., X thousths., C thousths.

51	+	2	+	4	+	0	+	7	+	0
8	+	9	+	3	+	1	+	4	+	6
42	+	3	+	0	+	9	+	2	+	4

or, the remainder is 42·30924.

70. Since ·06748 equals $\cdot 067\frac{48}{100}$, and since $\frac{48}{100}$ is less than $\frac{1}{2}$, therefore ·067 is a nearer approximation than ·068; the defect of the former is ·06748 — ·067 = ·00048, and the excess of the latter is ·068 — ·06748 = ·00052.

71. A whole number is divided by 1000 by placing a decimal point before that figure which stands in the third place from the right; for by this means the value of every figure is made 1000 times less. Thus, 8376 ÷ 1000 = 8·376. Similarly, it will appear that a whole number is divided by any power of 10, by cutting off as many figures for decimals as the divisor contains ciphers.

(*a*) This will serve to explain the rule for pointing in multiplication of decimals. Take as an example 52·36 × 4·275.

The multiplicand is $= \frac{5236}{100}$, the multiplier is $= \frac{4275}{1000}$; therefore the product is a whole number ÷ 100 times 1000, or ÷ 100000: which division will cut off as many figures, for decimals, as there are ciphers in 100000; that is, there will be as

many decimal figures in the product as there are in the two factors together.

(*b*) For pointing in division of decimals, the preceding explanation shews at once that we may subtract the number of decimals in the divisor from those in the dividend. But the best method, when the divisor is not a whole number, is first to multiply both dividend and divisor by that power of ten which is the understood denominator in the divisor. This will make the divisor a whole number, and is warranted by the principle that the quotient of two numbers is the same as the quotient of any equimultiples of those numbers. The proper pointing of the quotient is then obvious.

Suppose 2·4276 is to be divided by 28·56. The fractional part of the divisor is hundredths; therefore, multiplying both dividend and divisor by 100, we have

$$2856\,)\,242{\cdot}76$$

0·085 the quotient.

72. Cost of 54·625 yds. = £8·625

Do. 1 yd. $= \dfrac{£8625}{54625}$

Do. 14·25 yds $= \dfrac{£8625 \times 14{\cdot}25}{54625}$

The result is £2·25 = £2, 5s.

73. All vulgar fractions of which the denominator is divisible down to unity by 2 or 5, the factors of

10, are convertible into terminate decimals; such are the fractions $\frac{3}{16}$ and $\frac{7}{40}$.

Secondly, let D denote the given circulating decimal.

Since D is equal to $\frac{26}{100}+\frac{26}{10000}+$, &c. *ad inf.*

$\therefore$ 100 times D $= 26+\frac{26}{100}+\frac{26}{10000}+$, &c. *ad inf.*

Whence, by subtraction, 99 D $= 26$, or D $= \frac{26}{99}$.

74. To calculate 359 yards at 13s. 4d. a yard, is to find the product of 13s. 4d. $\times$ 359. But the Rule of Practice prescribes the easier method of resolving the given rate into such aggregate parts as are exact measures of a higher rate already calculated. Thus 13s. 4d. consists of 10s. and 3s. 4d.; the former of which is $\frac{1}{2}$ of a £, and the latter is $\frac{1}{3}$ of the former. Accordingly, we can say,—

359 yds. at £1	= £359	
Do. at 10s.	= $\frac{1}{2}$ of £359	= £179, 10s. 0d.
Do. at 3s. 4d.	= $\frac{1}{3}$ of £179, 10s.	= 59 16 8
359 yds. at 13s. 4d. amount to		£239 6 8

Or, we might have divided the given rate into two rates, each 6s. 8d., which is $\frac{1}{3}$ of a £, thus:—

£359 is the cost at £1

6s. 8d. = £$\frac{1}{3}$	119 13 4,	cost at 6s. 8d.
	119 13 4,	ditto.
	£239 6 8,	cost at 13s. 4d.

The general rule for resolving the given rate

into convenient parts is, to take from it at first the largest measure it supplies, then successively the next higher measures. Thus, 15s. $6\frac{3}{4}$d. may be resolved into 10s. + 5s. + 6d. + $\frac{3}{4}$d.; which are respectively equal to $\frac{1}{2}$ of a £, $\frac{1}{2}$ of 10s., $\frac{1}{10}$ of 5s., and $\frac{1}{8}$ of 6d.

The leading elementary principle in the Rule of Practice is, that the amount at a lower rate is the same fractional portion of the amount at a higher rate as the lower rate is of the higher. Thus, the amount at 6s. 8d. is $\frac{1}{3}$ of the amount at £1, because 6s. 8d. is $\frac{1}{3}$ of a £.

75. A per centage is a value allowed for every 100 that a given sum contains. Thus, 5 per cent. means 5 parts for every 100, or $\frac{5}{100}$ of some given quantity.

The calculation of 4 per cent. on £625 may be thus explained:—

£100 is to bear £4, loss or gain.

£1..... £$\frac{4}{100}$ „

£625..... £$\frac{4 \times 625}{100}$.

From which it appears that 4 per cent. on £625 is just $\frac{4}{100}$ of that sum, = £25; and that to find what sum calculated at 5 per cent. produces the same result, is obviously the same as to find what sum $\times$ 5 is = 625 $\times$ 4, viz. $\frac{625 \times 4}{5}$, or £500.

76. The fraction indicated by $3\frac{1}{3}$ per cent. is

$$\frac{3\frac{1}{3}}{100} = \frac{10}{300} = \frac{1}{30}.$$

Secondly, £$3\frac{1}{3}$ in every £$103\frac{1}{3}$ of the selling price was gain, $= 10$ in every 310; or $\frac{1}{31}$ of the selling price was gain, and the remaining $\frac{30}{31}$ of the selling price was the buying price.

77. Former population : present : : 100 : $102\frac{1}{2}$,
$= 200 : 205 = 40 : 41.$

78. (*a*) If £100 gain £$5\frac{1}{2}$, what does £280 gain?
(*b*) If £100 amount to £103, to what will £56 amount?
(*c*) If £60 lose £8, what does £100 lose?
(*d*) If £100 yield 7s. 6d., what sum yields 5s. 6d.?
(*e*) If 12s. 10d. per yard amount to £$102\frac{2}{3}$, what price per yard amounts to £100?
(*f*) If 5s. 9d. a yard amount to £$103\frac{1}{2}$, to what will 5s. 10d. a yard amount?
(*g*) If 7s. 4d. a yard amount to £$104\frac{1}{2}$, what price per yard amounts to £$106\frac{7}{8}$?

79. 5 per cent. per annum for 4 years is 20 per cent. altogether. Now, 20 per cent. in 5 years is $\frac{1}{5}$ of 20, or 4 per cent., per annum; 20 per cent. in 1 year is just 20 per cent. per annum; and 1 per cent. per annum becomes 20 per cent. in 20 years.

80. To simplify the problem, let us make the times uniform, by inquiring, If 8d. gain $4\frac{1}{2}$d. in 9 months, what does it gain in 6 months?

In 9 months the gain is $4\frac{1}{2}$d.

1 ..... $\frac{1}{2}$d.

6 .....3d.

Hence, if 8d. amounts to 11d. in 6 months, we can find what sum amounts to $13\frac{3}{4}$d. in the same time.

11d. is the amount of 8d.

1d. ..... $\frac{8}{11}$ d.

$13\frac{3}{4}$d..... $\frac{8 \times 13\frac{3}{4}}{11} = 10$d.

In this problem, the mistake to which pupils are liable is that of regarding the times as proportional to the amounts $12\frac{1}{2}$d. and $13\frac{3}{4}$d., whereas the times are proportional to the gains per penny. It would lead to a very erroneous result, if pupils, as is not unlikely, should perform the solution as follows:—

$12\frac{1}{2}$d. : $13\frac{3}{4}$d.
6mo. : 9 mo. } : : 8d. : $13\frac{1}{5}$d., which should be 10d.

81.

£1350

$2\frac{1}{2}$ p. c. $= \frac{1}{40} =$	33,	15s.
2 p. c. $= \frac{1}{50} =$	27	0
$4\frac{1}{2}$ p. c. $=$	£60,	15s.

Again: As $3\frac{1}{2}$ per cent. per annum for $2\frac{1}{2}$ years is $8\frac{3}{4}$ per cent. altogether, the second question may be proposed in this form: Calculate, by the method of Practice, $8\frac{3}{4}$ per cent. on £156.

£156

5 p. c. $= \frac{1}{20} =$ 7, 16s.
$2\frac{1}{2}$ p. c. $= \frac{1}{2}$ of 5 3 18
$1\frac{1}{4}$ p. c. $= \frac{1}{2}$ of $2\frac{1}{2}$ 1 19

$8\frac{3}{4}$ p. c. $= £13, 13s.$

82. (*a*) £7 in 4 years yields 17s. 6d. $= £\frac{7}{8}$.

£1 in 1 yr. ..... $\frac{£7}{8 \times 7 \times 4}$

£100 in 1 yr. ..... $\frac{£7 \times 100}{8 \times 7 \times 4} = £3\frac{1}{8}$.

(*b*) £100 yields £5 in 1 year.

£1 ..... £1 in $\frac{1 \times 100}{5}$ yrs.

£15 ..... $£3\frac{3}{8}$ in $\frac{100 \times 3\frac{3}{8}}{5 \times 15} = 4\frac{1}{2}$ yrs.

(*c*) The whole gain per cent. is $4 \times 3\frac{1}{2} = £14$.
Hence, £114 is the amount of £100, principal.

£1 ..... $£\frac{100}{114}$ ...

£513 ..... $\frac{£100 \times 513}{114} = £450$.

83. The product by $3\frac{1}{2}$ is the interest on £230, at $£3\frac{1}{2}$ per £ per annum; and the subsequent multiplication by 7 finds the interest at that rate for 7 years.

The product by 7 is an equivalent principal for

1 year at $3\frac{1}{2}$ per cent.; and the subsequent multiplication by $3\frac{1}{2}$ finds the interest on that principal at £$3\frac{1}{2}$ per £.

84. Find the interest on £374, for 213 days, at 4 per cent. per annum.

Interest on £374 means sum to be paid for use of £374.

If the money is lent for a year, the interest = £4 for every £100 in £374; that is £$4 \times \frac{374}{100}$.

If lent for half a year, the interest will be $\frac{1}{2}$ of £$4 \times \frac{374}{100}$.

If for a day, the interest will be $\frac{1}{365}$ of ditto.

If for 213 days, it will be $\frac{213}{365}$ of £$4 \times \frac{374}{100}$ = £$\frac{374 \times 213 \times 4}{100 \times 365}$.

Hence the rule might be,—Multiply principal by number of days and by rate, and divide by 36500.

Usual rule more convenient—Multiply by number of days and by *twice* the rate, and divide by *twice* 36500.

Easier division by 73000. Doubling the rate often removes a fraction, as in $3\frac{1}{2}$ per cent.; converts 5 per cent. into 10, which cancels a cipher from the divisor.

85. The interest on £1 for the first year, at 5 per cent.,

is £1 $\times \frac{5}{100}$, or £$\frac{1}{20}$; and therefore the amount is £1$\frac{1}{20}$.

Now, the amount of any principal for 1 year at 5 per cent. is, similarly, 1$\frac{1}{20}$ of the principal, or 1·05 times the principal.

Hence, the amount of £1 at the end of the first year, becoming a new principal for the second year, may be multiplied by 1·05 to give the amount at the end of the second year, and so forth. Thus:

Amount of £1 in 1 yr. = £1 × 1·05

Do. 2 yrs. = £1 × 1·05 × 1·05

Do. 3 yrs. = £1 × 1·05 × 1·05 × 1·05

So that the amount of £1 for any number of years, at 5 per cent. per annum, compound interest, is £1 multiplied by that power of 1·05, which is denoted by the number of years; and, therefore, the amount of £75 for 4 years will be

$$£1 \times 1{\cdot}05^4 \times 75 = 1{\cdot}05^4 \times 75 \text{ pounds.}$$

86. The true present worth of a sum due at a future time is the money that, being now put out at the allowed rate of interest, would at the future time amount to the sum due at that time. Thus, £60 in hand is as good as £63 payable at the end of a year, if the use of money is worth 5 p. c.; for £60 at 5 p. c. would become £63 at the year's end; therefore £60 is called the present worth of £63 due in 12 months.

If a banker lends £63 for a year, he remunerates himself, at the time of lending, by retaining

a year's interest of £63; that is, if interest be 5 per cent., he deducts £3, 3s., giving the borrower £59, 17s.; whereas, on the principle of true discount, the deduction should be only £3. The banker's discount is the true discount increased by the interest of the true discount, viz., £3 + $\frac{1}{20}$ of £3.

87. As the amount of £100 at 5 p. c., for 9 months, is £103$\frac{3}{4}$, the discount for present payment of a debt of £103$\frac{3}{4}$, due in 9 months, is £3$\frac{3}{4}$.

Disc. on £103$\frac{3}{4}$ = £3$\frac{3}{4}$

Do. on 415 = 15

Do. on 1 sum = $\frac{15}{415}$ or $\frac{3}{83}$ of the sum.

88. The amount of £100 at 6$\frac{1}{4}$ p. c., for 3 yrs., is £118$\frac{3}{4}$

Pres. val. of £118$\frac{3}{4}$ = £100

Do. of 475 = 400

Do. of 1 sum = $\frac{400}{475}$ or $\frac{16}{19}$ of the sum.

89. The interest on £100, for 63 days is $\frac{63}{365}$ of £4.

Amount of £100 = £100 + £$\frac{252}{365}$ = £$\frac{36752}{365}$

Disc. on £$\frac{36752}{365}$ = £$\frac{252}{365}$

Do. on £1 = £$\frac{252}{36752}$

Do. on £88·356 = $\frac{£252 \times 88{\cdot}356}{36752}$

or nearly 6 flor. 6 mils.

90. The amount of £1, in 2 years, at 4 p. c. per. ann., comp. int., is £1 × $1{\cdot}04^2$ = £1·0816.

Pres. val. of £1·0816 = £1

Do. of £1 = £$\frac{10000}{10816}$

Do. of £207·30$\frac{2}{3}$ = £$\frac{10000 \times 207{\cdot}30\frac{2}{3}}{10816}$

$\frac{100 \times 62192}{32448}$ = £191. 13s. 4d.

91. The meaning is, that the 1st divided by the 2nd equals $\frac{5}{9}$, and the 3rd divided by the 2nd equals $\frac{6}{9}$; or, the 1st is $\frac{5}{9}$ of the 2nd, and the 3rd is $\frac{6}{9}$ of the 2nd. Therefore, the sum of the 2nd and 3rd is $1\frac{6}{9}$ of the 2nd, and $\frac{5}{9}$ is $\frac{1}{3}$ of $1\frac{6}{9}$.

92. The number representing B is, in one instance, 6, in the other, 9; the least common multiple of 6 and 9 is 18; therefore, take the terms of the ratio of A to B = 15 : 18, and those of B to C = 18 : 20, and we have the continuous proportional relation of A, B, and C, as 15, 18, and 20.

Hence, A is 15 parts of the whole, B 18, C 20; therefore, the sum of A and C is 35 parts, and the difference of A and B is 3 such parts; so that if the value of 35 parts is 140, that of 1 part is 4, and that of 3 parts is 12.

93. Suppose that I owe a merchant £100, payable in 10 months, and £68, payable in 15 months; at what time will a single payment of £168 discharge both debts?

The most usual method of determining the

equated time proceeds upon the principle of bankers' discount. Let R denote $\frac{1}{12}$ of the rate of interest $\div 100$; then, the discount on £100 for 10 months would be £100 $\times$ 10 $\times$ R, that on £68 for 15 months would be £68 $\times$ 15 $\times$ R, and, therefore, that on £168 should be (£1000 + £1020) $\times$ R, which is equal to £168 $\times \frac{2020}{168} \times$ R; so that the equated time is $2020 \div 168 = 12{\cdot}024$ months.

This method evidently requires no particular rate of interest to be specified or assumed, but only to multiply each payment by its time, and to divide the sum of the products by the sum of the debts. And, for practical purposes, the answer thus obtained will, generally, be near enough to accuracy, although not conformable to the principle of true discount.

Another method, commonly called Kersey's Rule, ascertains the true present worth of each debt, at 5 per cent., and then determines the time at which the sum of the present values will amount to the sum of the debts. Thus:—

Pres. val. of £100 for 10 months =	£96
Do. of 68 for 15 months =	64
£168	£160

therefore, £160 paid now would discharge both debts; and to find in what time £168 will do so, is to find in what time the interest on £160 will be £8; that time is exactly 12 months.

This method is alleged to have a disadvantage, in permitting the equated time to vary from day to day. Thus, in 6 months after the above debts are constituted, there is £100 due in 4 months, and £68 due in 9 months; and if we use these data according to Kersey's Rule, we shall obtain less than 6 months as the equated time,—that is, less than 12 months from the time the debts were originally constituted.

This apparent inconsistency, however, admits of easy explanation. If A now owes B £100, payable in 10 months, the debt is *now* constituted, by A actually or virtually holding the present worth, £96. We should estimate the debt by the description which the question gives of it, and not constitute it anew, by saying, after 6 months, that A then owes B £100, payable in 4 months; for that is not a full statement of the obligation under which A has been placed.

Malcolm's Rule, therefore, which is the only other method usually recognised, and which, *for two payments*, is generally received as correct, should not be deemed more consistent with equity than Kersey's; for the invariableness of the result by Malcolm's method is caused by a neglect of the particular dates at which the debts are constituted. Malcolm's principle is, that the interest of the earlier debt, for the time from its being due to the equated period, should be equal to the true discount of the later payment, for the time from the

equated period to the date at which it would be due. Supposing the equated time, for the payments proposed in the above example, to be t months after the earlier period of 10 months, then, according to Malcolm, the interest of £100 for t months should be equal to the true discount of £68 for 15—t months;—a correct principle, if we may assume the earlier debt to be contracted at the end of 10 months, and payable then, and the later debt to be contracted at the equated time, and payable 5 months later than the other. The solution requires a quadratic equation, and would give 12·009 months.

But Malcolm's method manifests unsuitableness when more than two payments and periods are proposed. First, we have to equate for two of the payments, then to take the sum of the two, and its time thus found, and equate with a third payment, &c. The final result will be found to vary according to the choice we make of two payments to begin with.

If we construct a problem so that the earlier payment shall be the sum of the present values of the two payments, the answer by Kersey's Rule will agree with that by Malcolm's. Thus:

A owes B £40, payable in 13 years 4 months, and £27, payable in 13 years 9 months; find the equated time for one payment of £67.

The following calculation shews that the earlier payment is the present worth of £67.

Pres. val. of	£40 for $13\frac{1}{3}$ yrs.	= £24
Do. of	27 for $13\frac{3}{4}$ yrs.	= 16
Do. of	£67	£40

The time for £40 to gain £27, at 5 per cent., is $13\frac{1}{2}$ years; which is also the result according to Malcolm, whose inquiry is, If £40 be due at a certain date, and £27 be due 5 months later, how long after the first date will the equated time be?

The interest on £40 for the time required, t months, is to be equal to the discount on £27 for $5—t$ months,

$$\text{or, } \frac{40\times5\times t}{100\times12} = \frac{27\times5\times\frac{1}{12}(5—t)}{100+\frac{5}{12}(5—t)}$$

whence, by a quadratic equation, t is found = 2 months, beyond 13 years, 4 months.

Example of Three Payments.

Suppose £160 to be due at the end of $13\frac{1}{3}$ years, £108 in $13\frac{3}{4}$ years, and £201 in $17\frac{2}{9}$ years. Find the equated time.

If, by Malcolm's Rule, we equate for the 1st and 2nd payments, we obtain $13\frac{1}{2}$ years as the time for one payment of £268; if we then equate for £268, payable in $13\frac{1}{2}$ years, and £201 in $17\frac{2}{9}$ years, we obtain 15 years as the time for one payment of £469, the amount of the three debts.

Kersey's Rule also gives 15 years, the problem being constructed to suit both theories; but while there is no variation of answer in Kersey's method,

whichever two of the payments may be first selected for equating, there would be found by the method of Malcolm a result different from 15 years, if we first equated with £160, due in $13\frac{1}{3}$ years, and £201 due in $17\frac{2}{3}$ years.

If then, Malcolm's Rule, in the case of two payments, appears more definite (we cannot say more equitable) than Kersey's, the latter is more definite than Malcolm's when more than two payments are concerned.

94. Multiply 50 + 7
by 50 + 7

Product by 50 = 50 times 50
+ 7 times 50
Product by 7 = 7 times 50
+ 7 times 7

Hence, the square of 57, viz. 3249, is made up of 50^2 + (*twice* 50 + 7) × 7; and, by this method of expressing a square number, we shew how the involution affects the tens and units apart, and how the reverse process of evolution may be performed.

Accordingly, in proceeding to extract the root of 3249, we readily perceive that it must be less than 60, of which the square is 3600; and more than 50, of which the square is 2500. The first part of the root,

	3249(50+7
	2500
100	749
7	
107	749

then, will be 50; and we have to resolve 3249 into 50^2 + (twice 50 + a number of units) × that number of units.

The remainder, after deducting 50^2, is 749; which, being divided by twice 50 as a partial divisor, shews that the units' figure cannot be higher than 7, and being divided by (twice 50 + 7) ascertains that the quotient 7 completes the square exactly.

To explain the method of pointing in the usual process of extracting the square root, it may be observed that the root of 100 is 10, of 10,000 is 100, of 1,000,000 is 1,000, &c.; and, therefore, that the root of any whole square number less than 100 must consist of only one figure, the root of one between 100 and 10,000 of two places of figures, &c. Accordingly, if a comma, or other mark, be placed before every second figure of a number, beginning with the units, and counting towards the left, the number of periods into which the figures are thus divided will correspond to the number of figures in the root.

95. The decimal or denary scale of notation is that which represents numerical values by means of a series of powers of *ten*, descending from left to right. Thus, employing instead of unity the equivalent expression 10^0, we have 8325 = 8 times 10^3 + 3 times 10^2 + 2 times 10^1 + 5 times 10^0; also 763·5248 = 7 times 10^2 + 6 times 10^1 + 3 times 10^0 + 5 times 10^{-1} + 2 times 10^{-2} + 4 times 10^{-3} + 8 times 10^{-4}.

This is the scale used in common arithmetic; the negative forms 10^{-1}, 10^{-2}, &c., denoting the decimal fractions $\frac{1}{10^1}$, $\frac{1}{10^2}$, &c.

96. The septenary scale of notation is that which has *seven* for its radix or basis, and represents numerical values by means of a series of powers of *seven*.

The first twelve consecutive numbers in the septenary scale are, 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15; the expression 10 denoting 1 seven + 0 units, or seven; the next expression meaning 1 seven + 1 unit, or eight; and so forth.

97. In the quinary scale, the number *five* is expressed by 10; and 2244 in that scale denotes 2 times $(\text{five})^3$ + 2 times $(\text{five})^2$ + 4 times five + 4; which may be converted into an equivalent undenary expression by the following process:—

Resolving 4 fives into elevens, instead of tens, gives 1 eleven and 9 units. The square of 5 is 2 elevens and 3 units, which,

$$\begin{array}{rl} 4 \text{ units} = & 4 \\ 5 \times 4 = & 19 \\ \underline{5} & \\ 23 \times 2 = & 46 \\ \underline{5} & \\ 104 \times 2 = & 208 \\ \hline \textit{Ans.} & 275 \end{array}$$

multiplied by 2 gives 4 elevens and 6 units. In calculating the cube of 5, say, 5 times 3 units are 1 eleven and 4 units; write 4 and carry 1; 5 times 2 elevens are ten elevens, and 1 carried makes eleven elevens, or 1 square of

eleven and 0 over, &c. The answer is, 2 squares of eleven, with 7 elevens, and 5 units.

98. The nearest quinary square to 22, or 2 fives and 2, is 1 five and 4, the root of which gives the first quotient figure 3. In subtracting 4 from 2, borrow 5. Doubling the quotient figure, 3, gives 1 five and 1, for a trial divisor, which goes 3 times; then, 3 times 3 are 1 five and 4; write 4, and carry 1, &c. The root required is 33, that is, 3 fives and 3 units.

```
 22,44 (33
 14
 ---
113 | 344
    | 344
    ---
```

PART FOURTH.

SOLUTIONS OF PROBLEMS.

1. Cut away 1 quantity; left 5 times as much; therefore, stone was originally 6 times as much as the part cut away; or, 5 sixths of $7\frac{1}{2}$ cub. ft. is the reduced bulk.
2. Deducting 16 lbs. in every 112 lbs., or 1 seventh, of the gross weight, leaves 6 sevenths of it = 2 cwt. 46 lbs., so that 1 seventh of it will be 45 lbs. tare.
3. The buying price together with 2 ninths of the selling price will be the selling price; therefore, the buying price, £26, 19s., must be 7 ninths of the selling price, or 1 ninth of selling price is £3, 17s.
4. If the difference of the shares be 11 parts, N's share will be 5 such parts, and M's 16, and the sum divided will be 21; therefore, M has $\frac{16}{21}$, &c.
5. In one case $\frac{3}{5}$ of the number of shots succeed, in the other, $\frac{5}{8}$ of an equal number of shots; so that $\frac{3}{5} + \frac{5}{8}$, or $\frac{49}{40}$ of that equal number, amounts to 98; or 1 fortieth of it is 2 shots.
6. 16479·4 flor. at 25 francs for 10 flor. $= \frac{16479{\cdot}4 \times 25}{10}$ francs; this amount, at 11 flor. for 27 francs $= \frac{16479{\cdot}4 \times 25 \times 11}{10 \times 27}$ flor. $= \frac{£1647{\cdot}94 \times 55}{54}$ or $\frac{1}{54}$ more than the given sum.

7. If the proper rate was 1, the actual rate was $\frac{3}{5}$; therefore, if the proper time was 1, the actual time was $\frac{5}{3}$, since the whole journey is represented by $1 \times 1 = \frac{3}{5} \times \frac{5}{3}$. Accordingly, the actual time is $\frac{2}{3}$ longer than the proper time, or $2\frac{1}{2}$ hrs. is $\frac{2}{3}$ of the proper time; or, the journey should have been performed in $\frac{3}{2}$ of $2\frac{1}{2}$ hrs.

8. $7\frac{1}{2}$ lbs. sugar $= 8\frac{1}{4}$ soap, or 1 sugar $= \frac{11}{10}$ soap; $\therefore$ 13 sugar, or $5\frac{1}{2}$ coffee, $= \frac{11 \times 13}{10}$ soap, or 1 coffee $= \frac{13}{5}$ soap; $\therefore$ 5 coffee, or $1\frac{1}{2}$ tea, $= 13$ soap, or 1 tea $= 8\frac{2}{3}$ soap, or 4 lbs. tea $= 34\frac{2}{3}$ lbs. soap.

Otherwise, by the method of statement,

$$\left.\begin{array}{l} 7\frac{1}{2}\text{ Su}:13\text{ Su.} \\ 5\frac{1}{2}\text{ C}:\ 5\text{ C} \\ 1\frac{1}{2}\text{ T}:\ 4\text{ T} \end{array}\right\} :: 8\tfrac{1}{4}\text{ So.}:34\tfrac{2}{3}\text{ lbs. So.}$$

9. The fourth man gets 1 share, the third $1\frac{1}{6}$, the second $1\frac{11}{24}$, the first $2\frac{3}{16}$ shares; in all, $5\frac{13}{16}$ shares $=$ £8·37, or 93 shares $=$ £8·37 $\times$ 16, or 1 share $=$ £·09 $\times$ 16; &c.

10. Three classes compose $\frac{11}{20} + \frac{7}{24} + \frac{9}{80} = \frac{229}{240}$; therefore, the remaining $\frac{11}{240}$ of the population is 33, or, the whole is 720.

11. The rum is $\frac{11}{24}$ in the one, and $\frac{8}{18}$ in the other and these proportions are as 33 to 32.

12. 7 days $=$ the time of 2 men, or 1 day the time of 14 men, or $3\frac{1}{3}$ days the time of $4\frac{1}{5}$ men.

Now, 5 boys $=$ 2 men, or 3 boys $= 1\frac{1}{5}$ man; therefore, $4\frac{1}{5} - 1\frac{1}{5} = 3$ men.

13. Had the luggage all belonged to the first, he would have been charged, 35s.

viz. for his own previous excess 3s. 4d.
for the other passenger's do. 8 4
∴ for the other's exempted luggage 23 4
35s.

∴ exempted luggage added to each excess gives 26s. 8d. and 31s. 8d.

14. On the prime cost of 54 yds. he gains the prime cost of $7\frac{1}{2}$ yds.

" " 1 yd..... $\frac{15}{108}$ yd.

" " 100 yds..... $\frac{15 \times 100}{108}$ yds.

which is $13\frac{8}{9}$ per cent.

15. Every 104 in the selling price contains a profit of 4

" 1 ..... $\frac{4}{104}$

" 100 ..... $\frac{4 \times 100}{104}$

which is $3\frac{11}{23}$ p. c.

16. $10\frac{1}{2}$d. = £·04375 = 43·75 mils paid
Nearest whole no. = 44·00 received

43·75 gains ·25

∴ 1 gains $\frac{25}{4375}$; or 100 gains $\frac{2500}{4375}$

which is $= \frac{4}{7}$ per cent.

17. The retailer pays 108 for what originally cost 100.

20 p. c. = 21·6

Consumer pays 129·6 100

$\therefore$ 29·6 in every 129·6 is profit; $= \frac{296}{1296} = \frac{37}{162}$.

18. 20 lbs. increased by 4 p. c. $= \frac{20 \times 104}{100}$ lbs. sugar

3 lbs. lessened by 1 p. c. $= \frac{3 \times 99}{100}$ lbs. tea

$\therefore$ 20×104 lbs. sugar $= 3 \times 99$ lbs. tea

or, 1 lb. sugar equivalent to $\frac{3 \times 99}{20 \times 104}$ lbs. tea,

or, 26 lbs. sugar $= \frac{3 \times 99 \times 26}{20 \times 104}$ lbs. tea,

which is 3 lbs. $11\frac{2}{5}$ oz.

19. The amount of receipts would have continued the same had he sold $\frac{13}{16}$ of his previous quantity at 16d.; for 1 quantity at 13d. $= \frac{13}{16}$ of that quantity at 16d. But his receipts rose to $\frac{110}{100}$ of their previous amount; therefore, he sold $\frac{110}{100}$ of $\frac{13}{16}$, or $\frac{1430}{1600}$ of his previous quantity, which is a decrease of 170 on 1600, or $10\frac{5}{8}$ on 100.

20. On £100 the gain is 5 times the selling price per gall.;
on 20 shill. ,, $\frac{1}{20}$ of the same;
on 1 shill. ,, $\frac{1}{400}$ of the same;
on 25 shill. ,, $\frac{1}{16}$ of the selling price per gall.
$\therefore$ the selling price wanting $\frac{1}{16}$ of itself $=$ 25s;
or $\frac{15}{16}$ of the selling price $=$ 25s.
or the selling price is $\frac{25\text{s.} \times 16}{15} = 26\text{s. } 8\text{d.}$

21. 1 hat in 7 months $= 5\frac{1}{7}$ hats in 36 months,
1 „ 13 „ $= 2\frac{10}{13}$ „ „
$5\frac{1}{7}$ hats at 10s. ..... 51·4286s.
(36 + 29 + 22 + 15 + 8 + 1) mo.
111 mths. int. on 10s. ..... 4·625
——— 56·0536s.
$2\frac{10}{13}$ hats at 18s. ..... 49·8462
(36 + 23 + 10) mo.
69 mths. int. on 18s. ..... 5·175
——— 55·0212

The 18s. hat cheaper by ..... 1·0324s.
which is 1s. $0\frac{1}{2}$d. nearly.

22. Principal $\times 1{\cdot}04^3$ gives amount at comp. int.
do. $\times 1{\cdot}12$ ..... simp. int.
$\therefore$ do. $\times (1{\cdot}04^3 - 1{\cdot}12) =$ £1·9
£1·9 $\div$ ·004864 = £390·625.

23. The train and the sound begin to travel at the same time;
Train passes over $2\frac{1}{4}$ miles in $3\frac{3}{8}$ min.
Sound ..... $0\frac{99}{565}$

Sound precedes train by $3\frac{903}{45520}$ min.

24. $123\frac{171}{623}$ grs. chain gold sold for 20s.
1 gr. do. ..... $\frac{623}{38840}$ s.
1 oz. do. ..... $6\frac{23}{8}$ s.
value of chain gold $= \frac{18}{22}$ or $\frac{9}{11}$ of coinage do.
but the chains are sold for $\frac{11}{11}$ of do.
$\therefore$ charge for workmanship $= \frac{2}{11}$ of do.
$\frac{2}{11}$ of $6\frac{23}{8}$ s. = 14s. $1\frac{10}{11}$d.

25. No. favourable = 1 amount
No. unfavourable = 1 such amt. — 26

350 votes = 2 such amts. — 26
$\therefore (350 + 26) \div 2 = 188$ favourable.

26. Brother gets 1 share
Sister gets 1 such share — ·55 flor.

12·3 flor. = 2 such shares — ·55 flor.
$\therefore (12{\cdot}3 + {\cdot}55) \div 2 = 6{\cdot}425$ flor. to the brother.

27. 2ce the sum of the length and breadth = 1076 yds.
$\therefore$ the sum of the length and breadth = 538 yds.
The length is 1 measure
the breadth is 1 such measure — 144 yds.

538 yds. = 2 such measures — 144 yds.
$\therefore (538 + 144) \div 2 = 341$ yds. long.

28. They pass each other with a velocity equal to the sum of the rates per hour; but when both run in the same direction P passes H with a velocity equal to the difference of their rates per hour; therefore,
If the rate of H is 1 quantity
that of P is 1 such quantity + 4 miles

$\therefore$ sum of the rates = 2 such quantities + 4 miles
$(80 - 4) \div 2 = 38$ mi. an hour by H.

29. The miles per hour which the stream runs are a quantity which added to the man's rate in still water makes 5 miles, and subtracted from that rate makes $3\frac{1}{3}$ miles; therefore,

If the stream's rate is 1 quantity
that of the man is 1 such quantity + $3\frac{1}{3}$ mi.

sum of the rates = 2 such quantities + $3\frac{1}{3}$ mi.
$\therefore (5 - 3\frac{1}{3}) \div 2 = \frac{5}{6}$ mile per hour.

30. He rows 2 miles up in the time of 5 miles down,
or 10 miles up ..... 25 miles down,
or 10 mi. up and 10 down ..... 35 mi. down;
that is, he could row 35 mi. down in 5 hours,
or 7 mi. down in 1 hr. = 10 mi. down in $1\frac{3}{7}$ hr.
$\therefore 2\frac{4}{5}$ mi. up in 1 hour = 10 mi. up in $3\frac{4}{7}$ hrs.
If the stream's rate is 1 quantity
that of the man is 1 such quantity + $2\frac{4}{5}$ mi.

sum of the rates = 2 such quantities + $2\frac{4}{5}$ mi.
$\therefore (7 - 2\frac{4}{5}) \div 2 = 2\frac{1}{10}$ mi. an hr. by the stream.

31. Their sum is = 2ce the less + their difference; or,
their sum is = 15 + $\frac{5}{13}$ of their sum;
$\therefore \frac{8}{13}$ of their sum is 15; or their sum is $24\frac{3}{8}$;
$24\frac{3}{8} - 7\frac{1}{2} = 16\frac{7}{8}$, the greater.

32. Their sum is = 2ce the less + their difference; but
their difference is $\frac{32}{40}$, or $\frac{4}{5}$ of their sum.
$\therefore$ their sum is = 2ce the less + $\frac{4}{5}$ of their sum;
or, 2ce the less = $\frac{1}{5}$ of their sum, hence, their sum
is 10 times the less number; $\therefore$ the greater no. is
9 times the less, or, less : greater : : 1 : 9
$40 \times 9 = 360$, the sum of the numbers,
$360 \div 10 = 36$ the less number.

33. The actual rate upward, as compared with the rate
down, is less by $1\frac{3}{5}$ of the force of the midstream.

But they go down at 12 mi. and up at 8 mi. an hour;

$\therefore$ 4 miles $= 1\frac{3}{5}$ of the midstream's velocity, or $2\frac{1}{2}$ mi. per hour is the rate of the middle current; 12 — twice $2\frac{1}{2} = 7$ mi. an hour against the midstream.

34. 12 hrs. of A = 15 hrs. of B

$\therefore$ 12 hrs. of B + 12 hrs. of A = 15 hrs. of B + 12 hrs. of B;

that is, 12 hrs. of A and B = 27 hrs. of B

12×15 hrs. of A and B $= 27 \times 15$ hrs. of B

$\frac{12 \times 15}{27}$ hrs. of A and B = 15 hrs. of B;

or, the whole work by A and B in $\frac{12 \times 15}{27}$ hours.

35. 12 days of A + 12 days of B = 21 days of A

$\therefore$ 12 days of B = 9 days of A

12×21 days of B = 9×21 da. of A

$\frac{12 \times 21}{9}$ days of B = 21 da. of A;

or, the whole work is done by B in $\frac{12 \times 21}{9}$ days.

36. This question might be solved by first finding the time of E and F, and secondly, the time of E + F and G; thus,

$21 + 24 : 21 :: 24 : 11\frac{1}{5}$ hrs. by E and F together,

$11\frac{1}{5} + 28 : 11\frac{1}{5} :: 28 : 8$ hrs. by E, F, and G.

But the following is the usual method:—

E, F, and G, do $\frac{1}{21} + \frac{1}{24} + \frac{1}{28} = \frac{1}{8}$ of the work per hour; or the whole work in 8 hours.

37. F does $\frac{1}{3}^{1}$ of the work in a week; G and H do $\frac{2}{7}^{0} + \frac{1}{5}^{4}$ of it; therefore, F's ability is to that of G and H in the ratio of $\frac{1}{3}^{1}$ to $\frac{1}{3}^{9}\frac{8}{5}$, or 385 to 594, which is as 35 : 54.

38. M can reap $10\frac{1}{2}$ ac. in $2\frac{1}{4}$ hrs. $\times 7$, or $15\frac{3}{4}$ hrs.
N ..... $2\frac{1}{2}$ hrs. $\times 7$, or $17\frac{1}{2}$ hrs.
M and N do $\frac{4}{6}\frac{}{3} + \frac{2}{3}\frac{}{5} = \frac{20+18}{315}$ per hr.
or 38 times the whole in 315 hrs.,
or the whole in $8\frac{1}{3}\frac{1}{8}$ hrs.
Also, M should get $\frac{2}{3}\frac{0}{8}$ of 10s. 6d. = 5s. $6\frac{6}{19}$d.
N, the remaining $\frac{1}{3}\frac{8}{8}$ of 10s. 6d. = 4s. $11\frac{13}{19}$d.

39. In 3 days W does $\frac{3}{1}\frac{}{6}$, so that $\frac{1}{1}\frac{3}{6}$ are done by W and R together.
W and R do $\frac{1}{1}\frac{}{6} + \frac{1}{2}\frac{}{3} = \frac{3}{3}\frac{0}{6}\frac{}{8}$ in 1 day,
or 39 times the whole in 368 days,
or 13 times the whole in $122\frac{2}{3}$ days,
or $\frac{1}{1}\frac{3}{6}$ of the whole in $7\frac{3}{8}$ days.

40. One party does $\frac{1}{3}\frac{}{5}$ per day, the other only $\frac{1}{4}\frac{}{0}$; therefore, $\frac{1}{3}\frac{}{5} - \frac{1}{4}\frac{}{0} = \frac{1}{2}\frac{}{8}\frac{}{0}$ is the work of 3 men per day;
$\therefore \frac{2}{2}\frac{8}{8}\frac{0}{0}$ is the work of 840 men in 1 day,
or of 24 men in 35 days, or of 21 men in 40 days

41. D in 6 hrs. does $\frac{2}{7}$; E in 2 hrs. does $\frac{1}{1}\frac{}{1}$;
$\frac{2}{7} + \frac{1}{1}\frac{}{1}$ + F's work for 3 hrs. = $\frac{1}{2}$ the work;
$(\frac{1}{2} - \frac{2}{7}\frac{9}{7}) \div 3 = \frac{1}{4}\frac{9}{6}\frac{}{2}$ by F per hour.
$\frac{1}{2}\frac{}{1} + \frac{1}{2}\frac{}{2} + \frac{1}{4}\frac{9}{6}\frac{}{2} = \frac{3}{2}\frac{1}{3}\frac{}{1}$ by the three, per hour,
or 31 times the whole work in 231 hours,
or half the work in 231 $\div$ 62, hours.

42. A does a certain amount of work in 2 hours, B the same in 6 hours, C the same in 9 hours.

A, B, and C, do $\frac{1}{2} + \frac{1}{6} + \frac{1}{9} = \frac{7}{9}$ of it per hour, or the amount itself in $1\frac{2}{7}$ hour.

$\therefore$ 6 hrs. of B $= 1\frac{2}{7}$ hour of A, B, and C;

or A, B, and C, take $\frac{3}{14}$ of B's time;

$\therefore \frac{1}{14}$ of B's time $= 9$ hrs., or B's time $= 11\frac{5}{7}$ hrs.

43. 1 lb. composition burns $\frac{2}{3}$ of the time of $1\frac{1}{4}$ lb. wax;

or, $\frac{2}{3}$ of $1\frac{1}{4}$ of the time of 1 lb. wax;

i.e. 1 lb. comp. lasts $\frac{5}{6}$ as long as 1 lb. wax;

$\therefore \frac{5}{6}$ of 2s. 6d. $=$ 2s. 1d. per lb. for the comp.

44. 10 men and 2 boys are equal to 7 men and 10 boys.

$\therefore$ 8 boys $=$ 3 men, or 7 boys $= 2\frac{5}{8}$ men;

time for 7 boys, or $2\frac{5}{8}$ men, $=$ 12 hours,

$$\text{time for 1 man} = \frac{12 \times 21}{8} \text{ hours,}$$

$$\text{time for 8 men} = \frac{12 \times 21}{8 \times 8} \text{ hours.}$$

45. From one selling price to the other there is a decrease of 9d. $- 6\frac{1}{2}$d. $= 2\frac{1}{2}$d. per yard; also from the amount of one selling price to that of the other there is a decrease of 7s. 8d. + 1s. 11d. which is 115d.

$2\frac{1}{2}$d. is the decrease on the price of 1 yard,

1d..... $\frac{2}{5}$ yard.

115d..... $\frac{2 \times 115}{5}$ yds.

$\therefore$ the quantity is 46 yards.

7s. 8d. $\div$ 46 $=$ 2d. a yard gain;

9d. $-$ 2d. $=$ 7d. a yard prime cost.

46. By each poor person receiving 4d.—$2\frac{1}{2}$d., that is, $1\frac{1}{2}$d. less, the man had to give away 14d. + $5\frac{1}{2}$d., that is, $19\frac{1}{2}$d. less on the whole.

$1\frac{1}{2}$d. is the decrease for 1 poor person.

$\therefore$ $19\frac{1}{2}$d. is the decrease for 13 poor persons.

47. A deduction of 7d. in every 240d. reduces an income to $\frac{233}{240}$ of its gross amount.

Now, the gross incomes are, respectively, $3\frac{1}{4}$ and $3\frac{1}{2}$ hundredths of the principal; therefore,

$\frac{233}{240}$ of $(\frac{7}{200} - \frac{13}{400})$ of the principal = £$29\frac{1}{8}$;

or $\frac{1}{240}$ of $\frac{1}{400}$ of it = £$\frac{1}{8}$; therefore, the whole is £12,000.

48. Every 240d. in the given income is reduced to 233d., making it worth $\frac{233}{240}$ of its gross amount; also, every 240d. in the required income is reduced to 235d., making it worth $\frac{235}{240}$ of its gross amount; therefore, 233 parts of the former = 235 such parts of the latter;

$\frac{233}{235}$ of £151, 5s. $7\frac{1}{2}$d. = £149, 19s. $10\frac{1}{2}$d.

49. A gross income of £3 is derived from £100 stock or nominal capital. That gross income is reduced by the tax to $\frac{233}{240}$ of £3. And it is required to find the investment, or market price, of which that net income will be $3\frac{1}{3}$ per cent.

£$3\frac{1}{3}$ is made by investing £100.

£1..... £30.

$\frac{233}{240}$ of £3..... ... $\frac{233}{240}$ of £90.

$$\therefore \frac{£90 \times 233}{240} = £87\tfrac{3}{8}.$$

50. £4800 paid, at £80 for every £100 of stock, buys £6000 stock.

£6000 in the 4 p. cts., yields £240, first year.

£240 buys £300 additional stock;

£6300 yields £252 = £315 stock, second year.

£6615 yields £264, 12s. the third year.

51. The interest of the given present worth for 1 year is 9s. $3\frac{3}{7}$d., and for two years would be 18s. $6\frac{6}{7}$d.; therefore, subtracting the latter sum,

P. W. of £15, 4s. $3\frac{3}{7}$d. for 2 yrs., is £14, 5s. $8\frac{4}{7}$d.

multiplying both of these by 7 we have

do. of £$106\frac{1}{2}$,..... £100.

do. £1,..... £$\frac{200}{213}$.

do. £$14\frac{1}{5}$,..... £$\frac{200 \times 71}{213 \times 5}$.

52. Calling the prime cost 100,

Pres. value of selling price for 6 months,	102
Six months' int.	2·55
Selling price,	104·55
Pres. val. of selling price for time sought,	97·75
Int. of 97·75 for time sought,	6·80

Hence, the time sought will be found as follows:—

5 is the int. of 100 for 12 months,

1..... 1 .... 240 mo.

$\therefore$ 6·8 ..... 97·75 ... $\frac{240 \times 680}{9775}$ mo.

= 16 $\frac{16}{23}$ months.

53. 1*st.* 100 is the pres. val. of 100 + $\frac{5}{12}$ of 4:
or, $1\frac{2}{3}$ is the disc. of $101\frac{2}{3}$;
or, £100 is the disc. of £6100, due in 5 mo.
2*nd.* 100 is the pres. val. of 100 + $\frac{6}{12}$ of 3;
or, £100 is the pres. val. of £104, due in 16 mo.

Therefore, the disc. of 6100 equals the pres. val. of 104, the relative whole being 6204; accordingly, $\frac{6100}{6204}$ and $\frac{104}{6204}$ of £423, will give £$415\frac{10}{11}$, and £$7\frac{1}{11}$ respectively.

54. When the transfer is made, the mixture in each vessel will be 8 parts brandy, and 10 water; therefore, divide the capacity of each cask into 2 parts, as 8 to 10, and there will be,—
$\frac{10}{18}$ of 8 gall. water, transferred to the 8 gall. cask,
$\frac{8}{18}$ of 10 gall. brandy,..... 10 gall. cask,
each transfer = $4\frac{4}{9}$ gall.

55. When 3 men work in one set,
the other can do the work of 5 men;
∴ 2 in one set make the other do that of $3\frac{1}{3}$ men.
∴ the earnings of 7 men are reduced from 7 to $5\frac{1}{3}$, or, reduced by $1\frac{2}{3}$ on $7 = \frac{5}{21}$.

56. Given 5 hours, the time in which the hour hand has passed over $4\frac{5}{6}$ hour spaces, to find the time in which it will have passed over 5 hour spaces.
$4\frac{5}{6}$ spaces passed over in 5 hours.
or, 1 do..... in $1\frac{1}{29}$ hr.
∴ 5 do..... in 5 hrs. $10\frac{10}{29}$ min.

57. Call the hour hand H, and the minute hand M.
M passes over 12 hour spaces in the time H passes

over 1 hour space; or, M gains on H 11 hour spaces per hour.

Now, at 9 o'clock, H is 9 hour spaces in advance of M; therefore, in order that the hands may be 6 spaces apart, M must gain upon H 3 spaces; also, that they may be three spaces apart, M must gain six; and that they may be together, M must gain 9.

11 spaces gained in 1 hour,
1 do..... $\frac{1}{11}$ hour,
3 do..... $\frac{3}{11}$ hr. $= 16\frac{4}{11}$ min. past 9.
6 do..... $\frac{6}{11}$ hr. $= 32\frac{8}{11}$ do.
9 do..... $\frac{9}{11}$ hr. $= 49\frac{1}{11}$ do.

58. The minute hand, M, passes over 12 hour spaces in an hour, and the hour hand, H, passes over 1 such space in an hour; so that M gains on H 11 spaces per hour.

Now, at 5 o'clock, H is 5 spaces in advance of M, and, therefore, they will be at an interval of $\frac{13}{5}$, or, $2\frac{3}{5}$ hour spaces, when M has gained $2\frac{2}{5}$ spaces, and again, when M has gained $7\frac{3}{5}$ spaces on H.

11 spaces gained in 1 hour,
1 do..... $\frac{1}{11}$ hour
$2\frac{2}{5}$ do..... $\frac{12}{55}$ hr. $= 13\frac{1}{11}$ min. past 5.
$7\frac{3}{5}$ do..... $\frac{38}{55}$ hr. $= 41\frac{5}{11}$ do.

59. There will have been 2 occurrences of the interval when M has gained 12 hour spaces on H; the

first, before M overtakes H; the second, after M has passed H.

Therefore, there will have been 8 occurrences of the interval when M has gained 48 spaces on H, and the eighth occurrence will be when H is 10 hour spaces before M; hence the ninth time will be when M gains 8 spaces additional.

11 spaces gained in 1 hr. = 56 spaces in $5\frac{1}{11}$ hrs.

$5\frac{1}{11}$ hrs. after 10 = $5\frac{5}{11}$ min. past 3.

60. Whatever interval the hour hand, H, passed over, the minute hand, M, must have passed over 12 times as much space in the same time; and had M gone forward to its original position, it would have passed over 13 times the original interval, and would have completed three rounds of the dial, or 36 hour spaces;

$\therefore$ 13 times the interval = 36 hour spaces,

or, the interval was $2\frac{10}{13}$ hour spaces.

We have to find, therefore, at what time between 4 and 5 o'clock the minute hand is $2\frac{10}{13}$ spaces in advance of the hour hand.

M points to 12 when H points to 4,

$\therefore$ M has to gain upon H $4 + 2\frac{10}{13}$ spaces;

11 spaces gained in 1 hour,

$\therefore$ $6\frac{10}{13}$ do..... $\frac{8}{13}$ hour = $36\frac{12}{13}$ min. past 4.

61. 3 leaps of H = 2 of G,

$\therefore$ 4 „ of H = $2\frac{2}{3}$ of G,

and 50 „ of H = $33\frac{1}{3}$ of G;

H takes $2\frac{2}{3}$ of G's leaps to 3 of G,

$\therefore$ G gains $\frac{1}{3}$ of a leap in 3 leaps, or $33\frac{1}{3}$ in 300.

62. In 60 min. the train goes 21 mi., or $8\frac{3}{4}$ mi. in 25 min. The train will travel the remaining $37\frac{1}{4}$ mi. in the same time that the locomotive must travel 46 mi., viz., in $37\frac{1}{4} \div 21 = \frac{149}{84}$ hrs.

46 miles in $\frac{149}{84}$ hrs. $= \frac{46 \times 84}{149}$ mi. per hour.

63. A first travels $3\frac{1}{4} \times 1\frac{1}{2} = 4\frac{7}{8}$ mi.; and then the remainder, $20\frac{1}{8}$ mi., is passed over by A and B conjointly at $5\frac{3}{4}$ mi. an hour; so that they meet in $20\frac{1}{8} \div 5\frac{3}{4} = 3\frac{1}{2}$ hrs. from the time B sets out.

∴ A has travelled 5 hrs. at $3\frac{1}{4}$ mi. an hr. $= 16\frac{1}{4}$ mi.

64. E goes 3 spaces per hour, C goes 4, and the whole journey is $7 \times 8\frac{2}{3} = 60\frac{2}{3}$ spaces; which, at 3 spaces per hour, will take $20\frac{2}{9}$ hrs. of E; and at 4 spaces per hour, will take $15\frac{1}{6}$ hrs. of C.

65. In 2 hrs. A is $5\frac{1}{2}$ miles in advance of B; B gains on A $\frac{1}{4}$ mile in 1 hr., or $5\frac{1}{2}$ mi. in 22 hrs.

∴ B goes 66 mi. at 3 mi. an hr. in 22 hrs.

C goes 66 mi. at $4\frac{1}{2}$ mi. an hr. in $14\frac{2}{3}$ hrs.

or, C sets out $7\frac{1}{3}$ hrs. after B.

66. 100d. is the pres. val. of $101\frac{2}{3}$d. at 4 mths., or of $102\frac{1}{2}\frac{1}{1}$d. at 7 mths., or of $103\frac{3}{4}$d. at 9 mths.

∴ The pres. val. of 1d. at 4 mths., 7 mths., and 9 mths., is $\frac{60}{61}$, $\frac{240}{247}$, and $\frac{80}{83}$d. respectively.

Sum, $\frac{3650540}{1250561}$d. = pres. val. of three equal sums of ..... 1d.

∴ 3650540d. = do. do. of 1250561d.

∴ 182527d = do. do. of 62528·05d.

= £260, 10s. 8·05d.

67. A rise of 10 p. c. on tea and 10 p. c. on sugar, = 10 p. c. on the price of both, 6s. = 7·2d.
With 40 p. c. more on sugar the rise is 12·0d.

∴ 40 p. c. on the sugar is ..... 4·8d.
40d. on 100d. = 1d. on $2\frac{1}{2}$d. = 4·8d. on 12d.
Hence, the sugar cost 12d. and the tea 5s.

68. A rise of 2 p. c. on mutton and 2 p. c. on beef, = 2 p. c. on the price of both, 23s. $11\frac{1}{2}$d. = $5\frac{3}{4}$d.
With $3\frac{1}{2}$ p. c. less on mutton the rise is $\frac{1}{2}$d.

∴ $3\frac{1}{2}$ p. c. on the mutton is ..... $5\frac{1}{4}$d.
$3\frac{1}{2}$d. on 100d. = 1d. on $28\frac{4}{7}$d. = $5\frac{1}{4}$d. on 150d.
Hence, mutton, 150d. ÷ 20 = $7\frac{1}{2}$d. a lb.
beef, $137\frac{1}{2}$d. ÷ 22 = $6\frac{1}{4}$d. a lb.

69. F performs $\frac{8}{135}$ of a revolution per hour, G $\frac{22}{135}$, H $\frac{29}{135}$. G gains on F $\frac{14}{135}$ of a revolution per hour, or a whole revolution in $\frac{135}{14}$ hrs.
H gains on F $\frac{7}{45}$ of a revolution per hour, or a whole revolution in $\frac{90}{14}$ hrs.
G and F would come together in any multiple of $\frac{135}{14}$ hrs.; and, similarly, H and F in any multiple of $\frac{90}{14}$ hrs.: therefore, the three come together again in the least common multiple of these times, viz., in $\frac{2700}{14}$ hrs. = $19\frac{2}{7}$ hrs.

70. A completes a revolution every $7\frac{1}{3}$ hrs., B every $6\frac{1}{9}$ hrs.; and C goes and returns along the diameter, which is $17\frac{1}{2}$ miles, every $5\frac{5}{6}$ hrs. Therefore, the three will be again together at the starting-point in the least common multiple of $\frac{132}{18}$, $\frac{110}{18}$, and $\frac{105}{18}$ hrs., viz., in $\frac{4620}{18}$ hrs. = $256\frac{2}{3}$ hrs.

71. The original quantity + 48 minutes' supply is discharged by 11 pipes in 48 minutes = 1 pipe in 528 min.

The original quantity + 98 minutes' supply is discharged by 6 pipes in 98 minutes = 1 pipe in 588 min.

$\therefore$ the orig. quant. + 48 min. supply,
$= \frac{528}{588}$ of (the orig. quant. + 98 min. supply,)
$\therefore \frac{60}{588}$ of the latter = 50 min. supply,
or the latter = 490 min. supply,
$\therefore$ the orig. quant. = 392 min. supply,
525 cub. ft. in 392 min. $= 1\frac{12}{56}$ cub. ft. per min.

72. (The original grass + 27 days' growth) of $7\frac{1}{2}$ acres by 22 oxen in 27 days; or, of 1 ac. by 1 ox in 79·2 da.

(The original grass + 21 days' growth) of $8\frac{3}{4}$ acres by 31 oxen in 21 days; or, of 1 ac. by 1 ox in 74·4 da.

$\therefore$ the orig. grass + 21 da. growth,
$= \frac{744}{792}$ of (the orig. grass + 27 da. growth,)
$\therefore \frac{48}{792}$ of the latter = 6 da. growth,
or, the latter = 99 da. growth,
$\therefore$ the orig. grass = 72 da. growth,
72 + 27 da. gro. of $7\frac{1}{2}$ ac. in 27 da. by 22 ox.
$\therefore$ 72 + 24 da. gro. of $6\frac{1}{4}$ ac. in 24 da. by 20 ox.

73. (The original grass + 5 wks. growth) of 6 ac. by 18 ox. in 5 wks.; or, of 1 ac. by 1 ox in 15 wks.

(The original grass + 9 wks. growth) of 21 ac. by 45 ox. in 9 wks.; or, of 1 ac. by 1 ox in $19\frac{2}{7}$ wks.

$\therefore$ the orig. grass + 5 wks. growth,
$= \frac{7}{9}$ of (the orig. grass + 9 wks. growth,)
$\therefore \frac{2}{9}$ of the latter $= 4$ wks. growth,
or, the latter $= 18$ wks. growth,
$\therefore$ the orig. grass $= 9$ wks. growth,
9 + 5 wks. gro. of 1 ac. by 1 ox in 15 wks.
$\therefore$ 9 wks. gro. in $9\frac{9}{14}$ wks.

74. (The original grass + 4 wks. growth) of 7 ac. in 4 wks.; or, of 1 ac. in $\frac{4}{7}$ week $= \frac{3}{5}\frac{2}{6}$ wk.
(The original grass + 5 wks. growth) of 8 ac. in 5 wks.; or, of 1 ac. in $\frac{5}{8}$ wk. $= \frac{3}{5}\frac{5}{6}$ wk.
$\therefore$ the orig. grass + 4 wks. growth,
$= \frac{3}{3}\frac{2}{5}$ of (the orig. grass + 5 wks. growth),
$\therefore \frac{3}{3\,5}$ of the latter $= 1$ wks. growth,
or, the latter $= 11\frac{2}{3}$ do.
$\therefore$ the orig. grass $= 6\frac{2}{3}$ do.
4 wks. for $6\frac{2}{3}$ + 4 wks. gro. of 7 ac.
$\therefore$ 10 wks. for $6\frac{2}{3}$ + 10 wks. gro. of $11\frac{1}{5}$ ac.

75. (The original grass + 3 wks. growth) of $6\frac{3}{4}$ ac. by 29 ox. in 3 wks.; or, of 1 ac. by 1 ox in $12\frac{8}{9}$ wks.
(The original grass + 7 wks. growth) of $9\frac{9}{2\,0}$ ac. by 23 ox. in 7 wks.; or, of 1 ac. by 1 ox in $17\frac{1}{2\,7}$ wks.
$\therefore$ the orig. grass + 3 wks. growth,
$= \frac{8}{1}\frac{7}{1\,5}$ of (the orig. grass + 7 wks. growth,)
$\therefore \frac{2}{1}\frac{8}{1\,5}$ of the latter $= 4$ wks. growth,
or, the latter $= 16\frac{3}{7}$ do.
$\therefore$ the orig. grass $= 9\frac{3}{7}$ do.
1 ox from 1 ac. in $12\frac{8}{9}$ wks. gets $9\frac{3}{7}+3$ wks. gro.
$\therefore$ 28 ox. from $7\frac{1}{5}$ ac. in 1 wk. get $3\frac{3}{4}$ wks. gro.

$\therefore$ 15 wks. growth = 4 wks. food;

or, the orig. grass is $\frac{11}{4}$ of what grew while they grazed; that is, $9\frac{3}{7}$ wks. is $\frac{11}{4}$ of the time they grazed; hence, $9\frac{3}{7} \times \frac{4}{11} = 3\frac{3}{7}$ wks.

76. Yearly savings, deducting for repairs, = £45.

Original cost of machine,	£200
Interest,	10
Cost at 1 year's end,	210
Refunded at ditto,	45
	165
Interest	8·25
Cost at 2 years' end,	173·25
Refunded at ditto,	45
	128·25

By proceeding in this way, it will be found at the end of the 5th year that £6·6029 remains to be refunded, so that £193·3971 of the £200 has been paid back.

Otherwise, by applying the Rules of Compound Interest and Annuities, we have the amounts of £200 and an annuity of £45, respectively, as follow:—

£200 × $1{\cdot}05^5$	= 255·2563
£45 × ($1{\cdot}05^5$ — 1) ÷ ·05	= 248·6534
Remaining to be refunded,	6·6029
	200·0000
Amount paid back,	£193·3971

Note. The latter method might be shortened by first finding the principal that would yield £45 per annum at 5 per cent., namely, £900; for, as the compound interest of this sum for 5 years at 5 per cent. would be the amount of an annuity of £45, we have

$(1{\cdot}05^5 - 1) \times (900 - 200) = £193{\cdot}3971.$

77. As the numbers in both schools are divided in the same proportion, the sum of the two numbers in S will be to the sum of the two numbers in T as the difference of those in S to the difference of those in T; that is, as 15 to 9, or 5 to 3. Therefore, the sum of the numbers in S and the sum of the numbers in T are, respectively, $\frac{5}{8}$ and $\frac{3}{8}$ of the joint amount. Accordingly, taking these fractions of 184, we find 115 pupils in S, and 69 in T.

We have now to divide 115 into two parts, one of which shall be greater than the other by 15; and we have also to divide 69 into two parts, one of which shall be greater than the other by 9.

$\frac{1}{2}(115 - 15) = 50$ boys in S.

$\frac{1}{2}(69 + 9) = 39$ boys in T.

78. 105 lb. of malt is 16 per cent. less than the weight of barley which yields it, or is ·84 of it; ∴ the weight of barley $= 105 \div {\cdot}84 = 125$ lb.

∴ $(125 \div 51\frac{1}{2})$ bushels barley make 105 lb. of malt; hence the bulk of malt $= \frac{125}{51\frac{1}{2}} \times \frac{103}{100} = \frac{250}{100}$ $= 2\frac{1}{2}$ bu.

79. If t hours denote the time in which the four men together do the whole work, then the whole work could be done by A and B in $1\frac{7}{8}$ of t hours,
by A and C in $1\frac{8}{9}$ do.
by B and C in $2\frac{1}{127}$ do.
So that, when the four men work together, A and B do $\frac{8}{15}$ of the work, A and C $\frac{9}{17}$, B and C $\frac{127}{255}$; or A and B do $\frac{136}{255}$, A and C $\frac{135}{255}$, B and C $\frac{127}{255}$; so that A does $\frac{135-127}{255}$ or $\frac{8}{255}$ more than B, and A and B together do $\frac{136}{255}$; consequently A does $\left(\frac{136+8}{255}\right) \div 2$ or $\frac{72}{255}$, B $\frac{64}{255}$, C $\frac{63}{255}$, and D the remaining $\frac{56}{255}$.

Accordingly, B could do the work in $\frac{72}{64}$, or $1\frac{1}{8}$, of the time A would require; so that $\frac{1}{8}$ of the time A would take $= 1\frac{3}{4}$ hr.; $\therefore$ A would do the work in $1\frac{3}{4} \times 8 = 14$ hrs., B in $\frac{72}{64}$ of 14, or $15\frac{3}{4}$ hrs., C in $\frac{72}{63}$ of 14, or 16 hrs., D in $\frac{72}{56}$ of 14, or 18 hrs.

80. I received £275 in lieu of £1000, which is 66d. per £. Now, I got 72d. per £ of A's debt, together with 56d. per £ of B's; and it appears it would have come to the same thing had I got 66d. per £ of each debt—namely, 6d. *less* per £ of A's, and 10d. *more* per £ of B's. Hence 6 times A's debt balances or equals 10 times B's, or B's $= \frac{3}{5}$ of A's, or both debts—viz., £1000 $= 1\frac{3}{5}$ of A's.

A, therefore, owed £5000 $\div 8 =$ £625,

B, £1000 $-$ 625 $=$ £375.

APPENDIX:

WORKED SPECIMENS OF EXAMINATION PAPERS SET BY THE SOCIETY OF ARTS AND THE GOVERNMENT EDUCATION DEPARTMENT.

SOCIETY OF ARTS GENERAL EXAMINATIONS.

Answers to Arithmetic Paper of 1874.

1. Find, by Practice, the amount of $360\frac{5}{8}$ stones of beef, at 6s. 4d.

At £1 per stone, the amount = £$360\frac{5}{8}$ = £360, 12s. 6d.

£360, 12s. 6d.

4s. 0d.	= $\frac{1}{5}$	72	2	6
1 4	= $\frac{1}{3}$	24	0	10
1 0	= $\frac{1}{4}$	18	0	$7\frac{1}{2}$

£114, 3s. $11\frac{1}{2}$d. (*Ans.*)

2. If 216 men could finish an excavation in 136 days, how many men could do the same in 187 days?

Fewer men are required for a longer time; hence,

$$187 : 136 :: 216 \text{ m.} : \frac{216 \text{ m.} \times 136}{187}$$

$$= \frac{216 \text{ m.} \times 8}{11} = 157\tfrac{1}{11} \text{ m.} \quad (\textit{Ans.})$$

3. When £24, 15s. 5d. is the simple interest on £138, 7s. 6d., what is the interest on £19, 2s. 6d.?

£$138\frac{3}{8}$: £$19\frac{1}{8}$ :: £24, 15s. 5d.;

or, £1107 : £53 :: 5945d. : $\frac{5945\text{d.} \times 17}{123}$

$= \frac{101065\text{d.}}{123} = 821\frac{2}{3}\text{d.} = 68\text{s. } 5\frac{2}{3}\text{d.}$ (*Ans.*)

4 When tea is worth 3s. 1d. and coffee 1s. $4\frac{1}{2}$d. per lb., how much coffee, along with $5\frac{1}{4}$ lb. of tea, would amount to a guinea?

21s. 0d.

$5\frac{1}{4}$ lb. tea, at 3s. 1d. = 16 $2\frac{1}{4}$

To find how much coffee at 1s. $4\frac{1}{2}$d. = 4s. $9\frac{3}{4}$d.

$57\frac{3}{4}\text{d.} \div 16\frac{1}{2}\text{d.} = 231 \div 66 = 21 \div 6 = 3\frac{1}{2}$ lb. (*Ans.*)

5. I bought a piece of land, containing 1 ac. 2 ro. 32 po., at the rate of £40 per acre; and I have since sold $\frac{3}{4}$ of an acre of it for £42, 10s.: at what rate per acre must I sell the remainder, that my gain on the whole may be 30 guineas?

$6\frac{4}{5}$ ro., bought at £10 a rood = £68

proposed gain, $31\frac{1}{2}$

£$99\frac{1}{2}$

3 ro. sold for $42\frac{1}{2}$

$3\frac{4}{5}$ ro. to be sold for £57

= £15 per rood = £60 per acre. (*Ans.*)

6. In what time would £57, 15s. 10d. yield £1, 8s. 6d., simple interest, at the rate of $3\frac{3}{4}$ per cent. per annum?

£57, 15s. 10d. : £100
£$3\frac{3}{4}$: £1, 8s. 6d. } :: 1 yr. : $\frac{100 \times 342 \times 4}{13870 \times 15}$ yr.

$= \frac{10 \times 18 \times 19 \times 4}{73 \times 19 \times 15} = \frac{240}{365}$ yr. = 240 da. (*Ans.*)

7. Find the amount, by compound interest, of £412, 7s. 6d. in 2 years, at $4\frac{1}{2}$ per cent. per annum.

	£412·375
$2\frac{1}{2}$ p. c. $= \frac{1}{40}$	10·309375
2 p. c. $= \frac{1}{50}$	8·2475
	430·931875
$\frac{1}{40}$	10·773297
$\frac{1}{50}$	8·618637
	450·323809

= £450, 6s. 5·714d. (*Ans.*)

8. Divide 11 guineas among A, B, and C; making B's share equal to $\frac{3}{4}$ of A's, and C's share equal to $\frac{3}{8}$ of the sum of A's and B's shares.

A's share is 1 quantity,
B's " $= \frac{3}{4}$ of that quantity,
C's " $= \frac{3}{8}$ of $1\frac{3}{4}$ of ditto.

$\therefore (1 + \frac{3}{4} + \frac{21}{32})$ of A's share = the sum of the three shares;
or, $\frac{77}{32}$ of A's share = 11 guineas.

21s. $\times$ 11 $\div \frac{77}{32}$ = 96s. = £4, 16s., A's.
$\frac{3}{4}$ of £4, 16s. = £3, 12s., B's.
$\frac{3}{8}$ of £8, 8s. = £3, 3s., C's. } (*Ans.*)

9. Find the value of $(\frac{1}{234} - \frac{1}{923})$ of $35\frac{1}{2}$ square yards.

$$\frac{71}{2} \text{ sq. yds.} \times \frac{923 - 234}{923 \times 234} = \frac{1}{2} \text{ sq. yd.} \times \frac{689}{234 \times 13}$$

$= \frac{53}{468}$ sq. yd. $= \frac{53}{52}$ sq. ft. = 1 sq. ft. $2\frac{10}{13}$ in. (*Ans.*)

10. The sum of £72, 10s. 11d. is divided among A, B, and C; of whom A receives £2, 0s. 3d., and B £21, 9s. 4d. Express in lowest terms that vulgar fraction of the whole which C receives; and find also that decimal fraction of B's money which is equal to A's money.

A gets	£2,	0s.	3d.	= 483d.
B	21	9	4	= 5152
C	49	1	4	= 11776
	£72,	10s.	11d.	= 17411d.

The G.C.M. of 11776 and 17411 is 23;
hence, $\frac{11776}{17411} = \frac{512}{757}$ of the whole to C. (*Ans.*)
Also, $\frac{483}{5152} = \frac{3}{32} = .09375$. (*Ans.*)

11. If $\frac{22}{29}$ of $\frac{31\frac{3}{4}}{23\frac{3}{8}}$ of $12\frac{1}{2}$ cwt. be worth £32, what is the worth of ·0390625 of a cwt.?

$$\frac{22}{29} \text{ of } \frac{87 \times 8}{187 \times 4} \text{ of } \frac{25}{2} \text{ cwt.} : .0390625 \text{ cwt.} :: £32;$$

$$£32 \times .0390625 = £1{\cdot}25 = £1\tfrac{1}{4} = £\tfrac{5}{4};$$

$$\text{hence, } £\tfrac{5}{4} \times \tfrac{29}{22} \times \tfrac{187}{87 \times 2} \times \tfrac{2}{25} = £\tfrac{1}{4} \times \tfrac{1}{2} \times \tfrac{17}{3} \times \tfrac{1}{5}$$
$$= 17\text{s.} \div 6 = 2\text{s. } 10\text{d.} \quad (\textit{Ans.})$$

12. State, in simplest terms, the ratio between a profit of 30 per cent., and a profit of $3\frac{1}{2}$d. per shilling.

$$\frac{30}{100} : \frac{3\frac{1}{2}}{12} = 30 \times 12 : 100 \times 3\tfrac{1}{2} = 36 : 35. \quad (\textit{Ans.})$$

13. Find, to three decimal places, the number whose cube root is equal to the square root of 6.

$(\sqrt{6})^3 = \sqrt{(6^3)} = \sqrt{216} = 14{\cdot}697$ *nearly*. (*Ans.*)

14. What sum would yield £11, 8s. 9d. more by compound than by simple interest, in 3 years, at 5 per cent. per annum?

If P denote the required principal, we have
the amount by comp. int. $= P \times 1{\cdot}05^3 = P \times 1{\cdot}157625$
the amount by simp. int. $= P \times 1{\cdot}15$

hence, $£11{\cdot}4375 = P \times .007625$;
or, $£91{\cdot}5 = P \times .061$; $\therefore$ $P = £91500 \div 61 = £1500$. (*Ans.*)

15. What has a piece of silk, containing 4 dozen yards, cost me per yard, if, by selling one half of it at a profit of $7\frac{1}{2}$ per cent., and the other half at a loss of $2\frac{1}{2}$ per cent., I shall obtain £13, 6s. 6d. for the whole?

I shall sell 24 yds. at 1·075 of the prime cost,
and 24 yds. at ·975 of do.

£13, 6s. 6d. = 24 yds. at 2·05 of do.

$$\therefore \text{prime cost} = \frac{3198\text{d.}}{24 \times 2{\cdot}05} = \frac{31980\text{d.}}{12 \times 41} = 65\text{d.} = 5\text{s. } 5\text{d.} \quad (\textit{Ans.})$$

16. What is the annual rate of interest, when the true present worth of £462, 13s., for 5 months, is £456?

Int. on £456 for 5 months = £6, 13s.; to find the int. on £100 for 12 months.

$$\left.\begin{matrix} 465 : 100 \\ 5 : 12 \end{matrix}\right\} :: £6, 13\text{s.} : £3\tfrac{1}{2} \text{ per cent.} \quad (\textit{Ans.})$$

17. A sold goods to B, at a profit of 4 per cent.; and B then sold them to C, at a profit of $2\frac{1}{2}$ per cent., for £453, 9s.: what did the goods cost A?

$$102\tfrac{1}{2} : 100 :: £453{\cdot}45 : \frac{£18138}{41} \text{ paid by B;}$$

$$104 : 100 :: \frac{£18138}{41} : \frac{£226725}{533}$$

= £425, 7s. 6d. *nearly*. (*Ans.*)

18. There are two sums of money, M and N, the latter being equal to $\frac{5}{8}$ of the former. They are both put out at interest; N at a rate which is equal to $\frac{3}{4}$ of the rate on M, and M for a period which is equal to $\frac{5}{8}$ of the time of N. The interests together are £111, 6s. What are they separately?

For every £8 which M contains, N contains £5; and for every 5 years in M's time, there are 8 years in N's. Hence, the products of principal and time are equal, and the interests are simply proportional to the rates; that is, M's interest is to N's as 4 to 3, or M's is $\frac{4}{7}$ of £111, 6s., viz., £63, 12s., and N's is £47, 14s. (*Ans.*)

19. An acre of pasture land is divided into three fields, so that 8 times the first, 9 times the second, and 10 times the third would be equal areas. How many square yards are there in each field?

The areas are as $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, or as 45, 40, and 36; sum, 121. Hence, 4840 sq. yds. $\div$ 121, or 40 sq. yds., multiplied separately by 45, 40, and 36, gives 1800, 1600, and 1440 sq. yds. (*Ans.*)

20. A wine merchant mingled together two quantities of spirits, respectively worth 14s. and 25s. a gallon, and produced a compound worth 18s. He added, however, 3 gallons of water to this compound, and thereby reduced the value to 16s. 6d. a gallon. What were the quantities first mingled?

The reduction of 1s. 6d. on each gallon of the original compound must have amounted to the same as 3 gallons of the diluted compound at 16s. 6d.; hence (16s. 6d. $\times$ 3) $\div$ 1s. 6d. = 33 gall. in the original compound.

We have now to find the quantities at 14s. and 25s. which will make a compound of 33 gallons at 18s.

The average price, 18s., is 4s. greater than the inferior price, and 7s. less than the superior. Therefore the quantities of the inferior and superior kinds must have been as 7 to 4; and hence,

$\frac{7}{11}$ of 33 = 21 gall. at 14s.
$\frac{4}{11}$ of 33 = 12 gall. at 25s. } (*Ans.*)

SOCIETY OF ARTS GENERAL EXAMINATIONS.

Answers to Arithmetic Paper of 1875.

1. How much greater in value is 365 times 535 guineas than 627 times 365 crowns?

The former is greater by 365 times (535 guin. — 627 crs.) = 365 times (11235 s. — 3135s.) = £405 × 365 = £147825. (*Ans.*)

2. If 195 labourers can excavate 50,000 cubic yards of earth in 30 days, how many such labourers could perform the same work in 50 days?

50 : 30 :: 195 lab. : 117 lab. (*Ans.*)

3. Compute, by Practice, the cost of 489 yards of cloth at 16s. 10½d. per yard.

			£489		
10s.	0d.	= ½	244	10	0
5	0	= ½	122	5	0
1	3	= ¼	30	11	3
0	7½	= ½	15	5	7½

£412, 11s. 10½d. (*Ans.*)

4. If the value of 63 quarters 7 bushels of wheat be 184 guineas, what quantity is worth £103, 10s.?

21s. × 184 : 2070s. :: 63 qu. 7 bu.

$$\therefore \frac{511 \text{ bu.} \times 2070}{21 \times 184} = \frac{73 \text{ bu.} \times 90}{3 \times 8} = 34 \text{ qu. } 1 \text{ bu. } 3 \text{ pks.} \quad (Ans.)$$

5. Find, by Practice, the amount of a yearly income, at the average rate of £2, 18s. 9⅗d. per day.

£365

2

730	
10s. = ½	182 10
8s. = ⅕	146 0
9⅗d = 1/10	14 12

(*Ans.*) £1073, 2s.

(*Otherwise.*)

365 at £2, 18s. 9⅗d.

= 73 at £14, 14s. 0d.

14

£1022

1/20 = 51 2

£1073, 2s. (*Ans.*)

6. Find the sum of $2\frac{2}{63}+3\frac{3}{14}+4\frac{4}{21}+5\frac{5}{18}$.

L.C.M. of denominators, 126;

$$14+\frac{4+27+24+35}{126}=14+\frac{90}{126}=14\frac{5}{7}. \quad (Ans.)$$

7. What is the true present worth of £1020, due $2\frac{1}{2}$ years hence, at $2\frac{1}{2}$ per cent. per annum?

Int. on 100 = $2\frac{1}{2}\times 2\frac{1}{2}=6\frac{1}{4}$;

$106\frac{1}{4}$: 100 :: £1020

4 4

425 408000 ÷ 425 = £960. (*Ans.*)

8. How much more is the compound than the simple interest on £580, for 2 years, at $3\frac{1}{2}$ per cent. per annum?

Amt. by comp. int. = £580 × $1{\cdot}035^2$ = £580 × 1·071225.

Amt. by simp. int. = £580 × 1·07

The former is greater by £580 × ·001225

= £·7105 = 14s. 2·52d. (*Ans.*)

9. Divide 26·169473 by ·05078.

2616947·3 ÷ 5078 = 515·35. (*Ans.*)

10. What decimal fraction of 3 weeks is 3 days $10\frac{1}{4}$ hours?

3 da. $10\frac{1}{4}$ hrs. ÷ 21 da. = $82\frac{1}{4}$ hrs. ÷ 504 hrs.

= 329 ÷ 2016 = 47 ÷ 288 = 47 ÷ (4 × 8 × 9)

= 1·46875 ÷ 9 = ·163194. (*Ans.*)

11. Simplify $\dfrac{12\frac{3}{5} \times (8\frac{5}{7} - 5\frac{7}{8})}{10\frac{3}{5} \div \sqrt{(10\frac{1}{9} + 8\frac{2}{3})}}$.

$8\frac{5}{7} - 5\frac{7}{8} = 3 + \dfrac{40-49}{56} = 3 - \frac{9}{56} = 1\frac{59}{56}$;

also, $\sqrt{(10\frac{1}{9} + 8\frac{6}{9})} = \sqrt{18\frac{7}{9}} = 1\frac{3}{3}$;

hence the given expression $= (63 \times \frac{59}{56}) \div (53 \times \frac{3}{13})$

$= \frac{63}{56} \times 13 = \frac{9}{8} \times 13 = 14\frac{5}{8}$. (*Ans.*)

12. In how many days will the simple interest on £409, 8s. 2d. be £1, 15s. 4d., at $2\frac{1}{2}$ per cent. per annum?

$$\left.\begin{array}{r} £2\frac{1}{2} : £1,\ 15s.\ 4d. \\ £409,\ 8s.\ 2d. : £100 \end{array}\right\} :: 365 \text{ da.}$$

$$\therefore \frac{365 \text{ da.} \times 424 \times 100}{5 \times 49129} = \frac{42400}{673} \text{ da.} = 63 \text{ da.} \quad (\textit{Ans.})$$

13. If 14 men or 23 women can cut down a crop in $9\frac{1}{4}$ days, in what time could 9 men and 9 women do the same?

23 wom. $=$ 14 men; $\therefore$ 9 wom. $= \dfrac{14 \times 9}{23}$ men;

$\therefore$ 9 men and 9 wom. $= \frac{207}{23} + \frac{126}{23} = \frac{333}{23}$ men.

Given, therefore, that 14 men require $9\frac{1}{4}$ days, to find the corresponding time for $\frac{333}{23}$ men.

$333 : 14 \times 23 :: 9\frac{1}{4}$ da. $\therefore \dfrac{7 \times 23}{18}$ da. $= 8\frac{17}{18}$ da. (*Ans.*)

14. What decimal must be added to 24, that the square root of the sum may be $4\cdot\dot{8}\dot{9}$?

The square of $4\frac{89}{99}$, or of $\frac{485}{99}$, is $\dfrac{235225}{9 \times 9 \times 121}$

$= 2904\cdot\dot{0}12345679\dot{} \div 121 = 24\cdot0001020304$ &c.;

hence the decimal is $\cdot000102030405060708091+$. (*Ans.*)

15. A grocer sells goods at the rate of 8 lb. for 9s., which he bought at the rate of 9 lb. for 8s.; what does he gain per cent.?

Prime cost per lb. $= \frac{8}{9}$ or $\frac{64}{72}$ of a shill.; selling price $= \frac{9}{8}$ or $\frac{81}{72}$ of a shill.; rate of gain, 81 — 64 on 64.

$$64 : 100 :: 17 : 26\tfrac{9}{16}. \quad (Ans.)$$

16. A room is 23 ft. 7 in. long, 15 ft. 5 in. wide, and $11\frac{1}{2}$ ft. high. What would be the cost of covering its walls with paper, 15 inches wide, at $7\frac{1}{2}$d. per yard, deducting for door, &c., $\frac{1}{6}$ of the whole area?

No. of sq. ft. to be covered $= \frac{5}{6}$ of 2 (23 ft. 7 in. + 15 ft. 5 in.) $\times 11\frac{1}{2} = \frac{5}{6}$ of (39×23) sq. ft., = area of paper; $\therefore$ length of paper $= \dfrac{39 \times 23 \times 5}{6 \times 1\frac{1}{4}}$ ft. $= \dfrac{13 \times 23 \times 4}{2 \times 3}$ yds. at $7\frac{1}{2}$d., which amounts to 65×23d. $= 1495$d. $= £6$, 4s. 7d. (*Ans.*)

17. A money-changer gives £2, 19s. 7d. for 17 coins of one sort, and 4 of another; and he gives the same for 5 of the former and 18 of the latter. What value is given for each coin?

In the second instance, the 12 coins fewer of one sort must be equivalent to the 14 coins more of the other. Therefore, 17 coins of the first sort + 4 of the second = 17 of the first + $4 \times \frac{6}{7}$ of the first; or $20\frac{3}{7}$ of the first = 59s. 7d. Hence the values, 2s. 11d., and 2s. 6d. (*Ans.*)

18. A person invests £5329, 10s. in 3 per cent. stock at $70\frac{1}{8}$, and, on the price rising to $72\frac{1}{4}$, sells out, and invests the proceeds in 4 per cent. stock. He thus increases his income by £3, 4s. At what price did he buy into the 4 per cents.?

$\dfrac{5329\frac{1}{2}}{70\frac{1}{8}} = \dfrac{42636}{561} = 76$ cents. at £3 = £228 = 1st income; $\therefore$ 2nd income = $£231\frac{1}{5}$, arising from cents. at £4; hence the no. of cents. in the latter stock = $231\frac{1}{5} \div 4$

$= 57\frac{4}{5}$ cents., bought for $£72\frac{1}{4} \times 76$; and hence the price of each cent. must have been $(72\frac{1}{4} \times 76) \div 57\frac{4}{5} = (289 \times 19 \times 5) \div 289 = 95$. (*Ans.*)

19. A draper bought a piece of silk, containing a number of yards equal to the number of pence which it cost him per yard. He sold the whole, at a profit of $12\frac{1}{2}$ per cent., for 14 guineas. Find the selling price per yard.

$112\frac{1}{2} : 100 :: £14,\ 14s. : 3136d.$, prime cost of the whole; $\sqrt{3136} = 56$ the no. of pence per yard, prime cost.

$100 : 112\frac{1}{2} :: 56d. : 63d. = 5s.\ 3d.$ (*Ans.*)

20. A and B set out at 8 o'clock A.M. from the same place in the same direction. A travels at the rate of 3 miles 1120 yards per hour, and after 20 minutes, turns and goes back as great a distance as B has travelled in the 20 minutes; he then turns again, and, pursuing his journey, overtakes B at 6 o'clock P.M. the same day. Find B's rate per hour.

When B is overtaken, each has travelled, in all, 10 hours; but A's whole number of miles exceeds B's by twice 20 minutes' walking of B; ∴ B would have taken 10 hrs. 40 min. to go the number of miles which A walked in 10 hrs. Hence A's rate is to B's as $10\frac{2}{3}$ to 10, or as 16 to 15;

$16 : 15 :: 3\frac{7}{11}$ mi. $: 3\frac{9}{22}$ mi., or 3 mi. 720 yds. (*Ans.*)

GOVERNMENT EDUCATION DEPARTMENT.

Answers to Higher Arithmetic Paper of 1874.

[*Algebra was allowed to be employed in solving questions in this Paper.*]

1. Multiply 2 ac. 1 ro. 14 per. by

$$\frac{\frac{4}{5}+\frac{6}{7}+\frac{13}{15}+\frac{10}{21}}{\frac{1}{5}+\frac{5}{7}+\frac{7}{15}+\frac{13}{21}}.$$

Here, multiplying the dividend and divisor by the L.C.M. of 15 and 21, which is 105, we have

$$\frac{84+90+91+50}{21+75+49+65}=\frac{315}{210}=\frac{3}{2}=1\tfrac{1}{2};$$

	ac.	ro.	per.	
2)	2	1	14	$\times\ 1\frac{1}{2}$
	1	0	27	
	3	2	1	(*Ans.*)

2. Divide 22 tons 8 cwt. 4 lb. by $1\frac{1}{19}$.

	tons.	cwt.	qr.	lb.	
20)	22	8	0	4	$\times\ \frac{19}{20}$
	1	2	1	17	
	21	5	2	15	(*Ans.*)

3. Find the vulgar fractions which are represented by $0\cdot\dot{7}69230\dot{}$ and $0\cdot\dot{8}3\dot{7}$; and determine the difference between the vulgar fraction and six places of the decimal which represents it in each case.

$$\frac{769230}{999999}=\frac{85470}{111111}=\frac{7770\times 11}{111\times 1001}=\frac{10}{13}. \quad (Ans.)$$

$$\frac{837}{999}=\frac{93}{111}=\frac{31}{37}. \quad (Ans.)$$

Hence, $\frac{10}{13}$, being equal to $\cdot769230\frac{10}{13}$, exceeds the value of the six decimals by $\frac{10}{13}$ of one millionth of unity, or by the millionth part of $\frac{10}{13}$. (*Ans.*)

Similarly, $\frac{31}{37}$, being equal to $\cdot 837837\frac{31}{37}$, exceeds the value of the six decimals by the millionth part of $\frac{31}{37}$. (*Ans.*)

4. The division of the sum of the digits of any number by 9 gives the same remainder as that obtained by the division of the number itself by 9. Prove this.

10, 100, 1000, &c.,—that is, every power of 10,—when divided by 9, leaves a remainder 1; therefore twice, thrice, four times, . . . eight times, any power of 10 will, when divided by 9, leave 2, 3, 4, ... 8, respectively. Hence, for example, 687, consisting of $600+80+7$, is an exact number of *nines* $+6+8+7$, or an exact number of *nines* + whatever remains after dividing $6+8+7$ by 9. And so for any other number.

5. If 14 ounces of amalgam formed of 21 parts of gold and 3 parts of silver be worth 27 ounces of amalgam formed of 10 parts of gold and 14 parts of silver, what is the relative value of gold and silver?

The amalgam of 14 ounces consists of $\frac{21}{24}$ of 14, or $12\frac{1}{4}$ oz. gold, and $1\frac{3}{4}$ oz. silver. The equivalent amalgam of 27 oz. consists of $\frac{10}{24}$ of 27, or $11\frac{1}{4}$ oz. gold, and $15\frac{3}{4}$ oz. silver. Accordingly, the value of $12\frac{1}{4}-11\frac{1}{4}$ oz. of gold = that of $15\frac{3}{4}-1\frac{3}{4}$ oz. of silver; or 1 oz. of gold = 14 oz. of silver; or the value of gold is to that of silver as 14 to 1. (*Ans.*)

Note.—If x be used to denote the number of times that gold is as valuable as silver, we have $12\frac{1}{4}$ oz. gold $= 12\frac{1}{4}\ x$ oz. silver, and $11\frac{1}{4}$ oz. gold $= 11\frac{1}{4}\ x$ oz. silver; $\therefore\ 12\frac{1}{4}\ x+1\frac{3}{4}=11\frac{1}{4}\ x+15\frac{3}{4}$, or $x=14$.

6. If a cubic foot of ice weighs 930 ounces avoirdupois, what is the pressure of a glacier 2000 feet deep on a square yard of the level surface on which it rests?

A sq. yd. = 9 sq. ft.; hence we have 9 cub. ft. × 2000, at 930 oz. per c. ft., = 16740000 oz. = 1046250 lb. = 9341 cwt. 58 lb. = 467 tons, 1 cwt. 58 lb. (*Ans.*)

7. If 3 men can earn as much in a day as 5 women, and 1 woman as much as 2 boys, and if 4 men, 7 women, and 6 boys working together can earn £25 in 10 days, what will be the earnings of 5 men, 3 women, and 4 boys working together for 12 days?

Since 3 men = 10 boys, ∴ 4 men = $13\frac{1}{3}$ boys, and 5 men = $16\frac{2}{3}$ boys;

Hence, 4 m. + 7 w. + 6 b. = $13\frac{1}{3} + 14 + 6 = 33\frac{1}{3}$ boys;

and, 5 m. + 3 w. + 4 b. = $16\frac{2}{3} + 6 + 4 = 26\frac{2}{3}$ boys.

$$\left.\begin{matrix} 33\frac{1}{3} : 26\frac{2}{3} \\ 10 : 12 \end{matrix}\right\} :: £25 : \frac{£25 \times 80 \times 12}{100 \times 10} = £24. \quad (Ans.)$$

8. Find the weight in ounces of a hollow globe of iron, whose external diameter is 2 feet, and internal diameter 1 foot 10 inches; given that 1 cubic foot of iron weighs 7788 ounces, and $\pi = 3{\cdot}1416$.

Volume of the whole sphere = $2^3 \times \frac{1}{6}\pi$;

Volume of hollow space = $(\frac{11}{6})^3 \times \frac{1}{6}\pi$;

cub. ft. of iron = $(8 - \frac{1331}{216}) \times {\cdot}5236$

$$= \frac{397}{216} \times {\cdot}5236 = \frac{397 \times {\cdot}1309}{54};$$

$$\therefore \text{wt. in oz.} = \frac{7788 \times 397 \times {\cdot}1309}{54} = \tfrac{1}{9}(1298 \times 397 \times {\cdot}1309)$$

$$= 7494{\cdot}84 \text{ oz.} \quad (Ans.)$$

9. What is the length of the edge of a cube whose solid contents are the same as a rectangular solid, the edges of which, meeting at any of its angles, are 154 ft. 11 in., 70 ft. 7 in., and 53 ft. 1 in.?

Solidity of the cube in inches = $1859 \times 847 \times 637$

$= 169 \times 11 \times 77 \times 11 \times 7 \times 7 \times 13 = 13^3 \times 11^3 \times 7^3$;

the cube root of which, namely, $13 \times 11 \times 7 = 1001$ in. = 83 ft. 5 in. (*Ans.*)

10. Find the rateable value of a parish which produces a rate of £7297, 8s. $10\frac{3}{4}$d. at 1s. 11d. in the pound.

7005547 farth. ÷ 92 farth. = £76147, 5s. (*Ans.*)

11. A piece of wire was cut into two lengths, which bore to one another the relation of $\frac{3}{4}$ to $\frac{4}{5}$; and the difference in their lengths was one inch. What was the length of the wire?

$\frac{3}{4} : \frac{4}{5} = 15 : 16$; ∴ the lengths were $\frac{15}{31}$ and $\frac{16}{31}$ of the whole; and their difference $\frac{1}{31}$ of the whole, = 1 inch; hence the length of the wire was 31 inches, or 2 ft. 7 in. (*Ans.*)

12. A pig, a cow, and a horse cost together £65. Their prices were in geometric ratio; and the price of the horse and cow together was three times the price of the pig and cow together. What were their prices?

Let P, C, and H denote the several prices of pig, cow, and horse, and r the common ratio; we have $C = rP$, and $H = rC$; ∴ $H + C = r(P + C)$; hence $r = 3$.

We have now $P + 3P + 9P = 13P = £65$; whence the price of the pig £5, of the cow £15, of the horse £45. (*Ans.*)

13. What sum of money will amount to £172429, 14s. 6d. in 4 years at compound interest, the interest for the 1st, 2nd, 3rd, 4th years being 4, 3, 2, 1 per cent. respectively?

£172429·725 ÷ (1·01 × 1·02 × 1·03 × 1·04)
= £17242972500000 ÷ 110355024 = £156250. (*Ans.*)

14. A person sold out £296000 in the 3 per cents at $91\frac{1}{4}$, paid as brokerage 5s. per cent. on the amount realized, and £54, 15s. in other expenses. He invested the remainder in two equal amounts, in New Zealand 5

per cents, and Queensland 6 per cents, and increased his yearly income by £5070. The price of the New Zealand stock was $102\frac{1}{2}$. What was the price of the Queensland stock?

Income from the 3 per cents = 2960 cents at £3 = £8880.

2960 cents sold at $£91\frac{1}{4}$ = £270100

brokerage $\frac{1}{4}$ of £2701 = £675, 5s.

other expenses = 54 15 730

2) 269370

Income from investing £134685

in New Zealand 5 per cents at $102\frac{1}{2}$

$= \frac{£269370 \times 5}{205} = £6570$; ∴ income from investing £134685 in Queensland 6 per cents = £8880 + £5070 − £6570 = £7380;

∴ price of Queensland stock $= \frac{134685 \times 6}{7380} = 109\frac{1}{2}$. (*Ans.*)

15. Simplify

$$\frac{\frac{22}{31}+\frac{3}{29}}{\frac{34}{41}+\frac{5}{19}} \div \frac{\frac{21}{37}+\frac{8}{23}}{\frac{29}{43}+\frac{13}{17}}$$

$$\frac{638+93}{31\times 29} \div \frac{646+205}{41\times 19} = \frac{731}{899} \times \frac{779}{851};$$

$$\frac{483+296}{37\times 23} \div \frac{340+559}{43\times 17} = \frac{779}{851} \times \frac{731}{899};$$

the 1st of these results ÷ the 2nd gives 1. (*Ans.*)

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